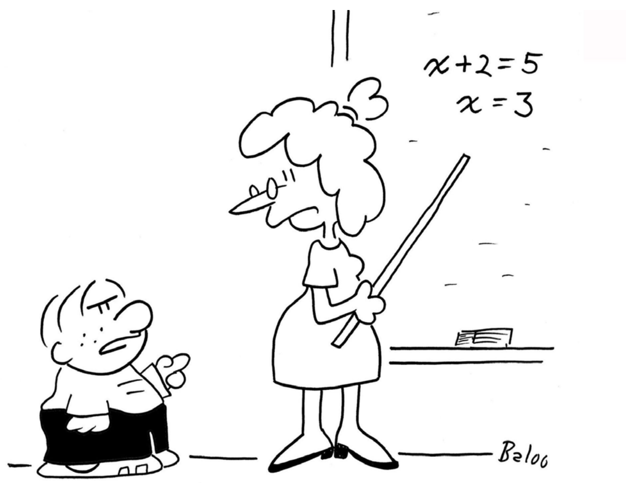
Chapter 1:

ALGEBRAIC THINKING



"Just a darn minute — yesterday you said that X equals **two**!"

In this chapter…

You might wonder why future elementary teachers should master algebra, a topic usually studied (by that name, anyway) in 8th grade and beyond. But the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics-Algebra at https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Principles,-Standards,-and-Expectations/ has algebra standards beginning in kindergarten as shown below.

In prekindergarten through grade 2 all students should…

- use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations;
- model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols;

In grades 3–5 all students should…

- represent the idea of a variable as an unknown quantity using a letter or a symbol
- express mathematical relationships using equations;
- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions

In the **Massachusetts Mathematics Curriculum Framework**…

Grade 3 - 3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.

> 8. Solve two-step word problems using the four operations for problems posed with whole numbers and having whole number answers. Represent these problems using equations with a letter standing for the unknown quantity.

Elementary school students already have some background in abstraction and generalization, the fundamental ideas in algebra. They are all capable of learning to formalize these ideas. Your job as an elementary school teacher will be to provide your students with even more experiences in abstraction and generalization in a mathematical context, so that these ideas will be more natural when they get to a class with the name "Algebra."

Pre-Class Work

Watch the two videos below about algebra and respond to the prompts that follow.

1. Algebra made Easy. Math concepts for kids: https://www.youtube.com/watch?v=OU87O69sTLM (~ 3 min.)

2. Algebra Basics. What is Algebra-Math Antics (Subscribe to this channel as we will be using various videos from it): https://www.youtube.com/watch?v=NybHckSEQBI (~12 minutes)

Prompts to respond to for pre-class work:

1. What did you learn about the origins of algebra? Explain in 2-3 sentences.

2. How is algebra different from arithmetic? Explain with an example.

3. What is the biggest power of algebra? How does it make our life better? Why do we need algebra? (There is no right or wrong answer here. Just share your thoughts!)

Alternate Pre-Class Work

Write a page on the history of algebra at the elementary level as if explaining it to a 4th grader (compare/contrast with arithmetic)

Why study Algebra?

Here are a few reasons:

1. Algebra is a tool for solving problems. Even when algebra is not necessary to solve a particular problem, using algebraic thinking can make it easier and in many cases, algebra is necessary to solve problems.

2. Algebra helps us think abstractly. It is a tool for thinking more deeply about operations like addition, subtraction, multiplication, and division.

3. Algebra helps us to understand and explain why operations work the way they do and to reinforce their properties.

1.1 Introduction to Algebraic Thinking

What is Algebra?

According to one source at

https://wagner.nyu.edu/files/students/Math_Review_-_Algebra_Operations.pdf

Algebra is a branch of mathematics that uses mathematical statements to describe relationships between things that vary.

For example, we could use a mathematical statement to describe the relationship between the width of a cube (length=width=height) and its surface area.

When we use a mathematical statement to describe a relationship, we often use letters (that are called “variables”) to represent the quantities that vary, since the values of these quantities change (depending on the width of the cube).

Example 1.1.1

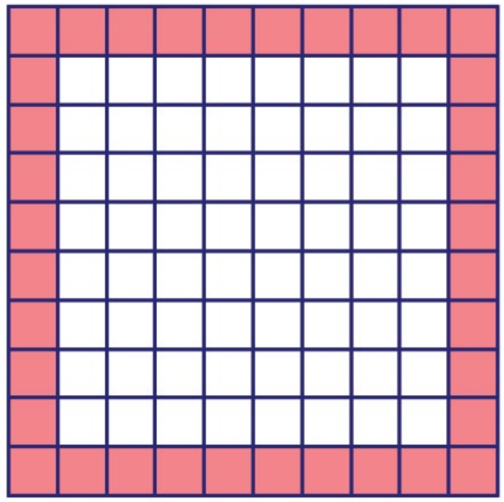
Determine the surface area for a cube with width 7 cm in two ways as follows.

Way 1: Using algebra (creating an equation that relates the width, w, to the surface area, S, and then using your equation find the answer). Be sure to define your variables clearly here.

Way 2 (to determine the surface area for a cube with width 7 cm): Using objects (handout cubes) arithmetic and pictures (as if you are presenting to a class that does not know algebra). Explain your process and show your work below.

Example 1.1.2

Below is a large square made up of 100 smaller unit squares (1 by 1). The unit squares along the border are shaded.



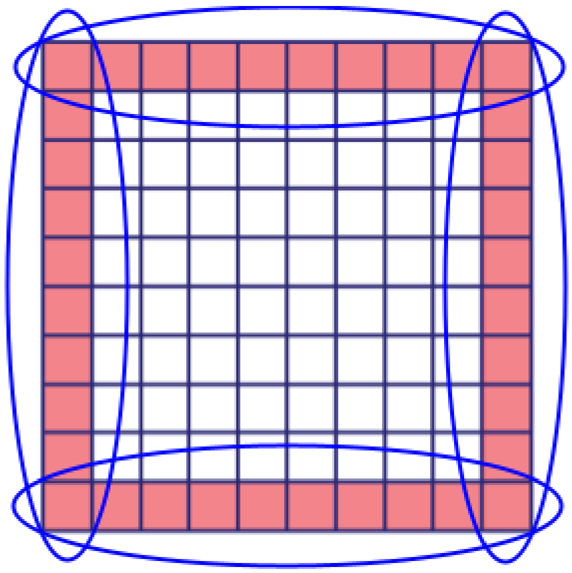
Without counting one by one, try to figure out how many shaded border squares there are in the picture. State your answer below and clearly describe how you found it. It might help to mark up the figure above to illustrate your approach. Then check your answer by counting the squares one by one.

Share your approach with other students in the class and if you come across a different approach, describe it below.

Taban calculated the number of border squares as $(10\ x\ 4)\ -\ 4$. He justifies his answer as follows:

Since the dimensions of the big square are 10 x 10, there are 10 squares along each of the four sides. So that gives me 40 shaded border squares. But then each corner is part of two different sides. I've counted each of the corners twice. So I need to make up for that by subtracting 4 at the end.

Taban showed the picture below to illustrate his explanation:



(10 x 4) - 4

- What do you think of Taban's solution? Are you convinced? Could he have explained it more clearly?

- How did Taban's solution compare/contrast with your own?

- Explain how Taban's shading and circling in his picture helps justify his answer.

Below are expressions that other students used to calculate the number of shaded squares along the border of a 10 x 10 square. For each expression, write a justification that explains why that expression represents the number of shaded border squares correctly. Do NOT modify the expression, as it represents the student's approach that you are trying to explain. It might be helpful to include a sketch of the square and illustrate your reasoning.

1. Valentino calculated $10 + 10 + 8 + 8$

2. Kayla calculated $4 \, x \, 9$

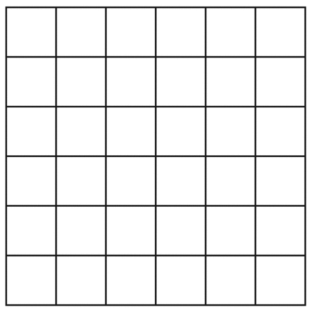
3. Lin calculated $(10 \, x \, 10) - (8 \, x \, 8)$

4. Marco calculated $(4 \, x \, 8) + 4$

5. Denzel calculated $10 + 9 + 9 + 8$

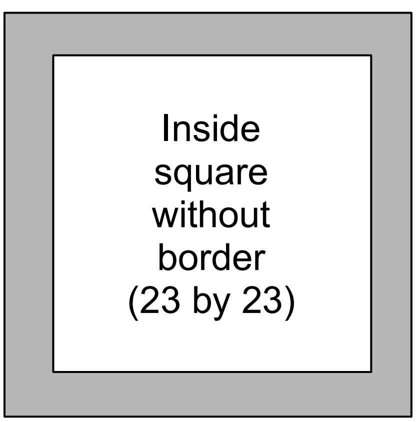
Example 1.1.3

Consider the 6 by 6 square below. Shade the border squares and use one of the techniques from example 1.1.2 to calculate the number of shaded border squares. Write an explanation and mark the picture below to illustrate your reasoning.



Example 1.1.4

Consider the 25 by 25 square with shaded unit squares along the border as shown below. Use one of the techniques from example 1.1.2 to calculate the number of shaded border squares. Write an explanation and mark the picture below to illustrate your reasoning.



Example 1.1.5

If you have an n by n square with shaded border squares, how many (unit) shaded border squares will there be? Justify your answer with words and a picture.

Example 1.1.6

a) Suppose you have 64 unit squares. Can you use all of these to make the border of a square? If yes, what are the dimensions of the square with the border? If no, why not? Explain your reasoning below.

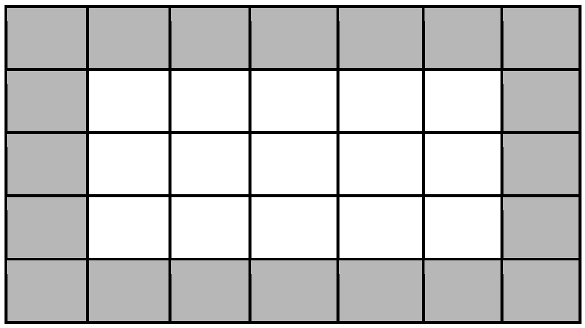
b) Answer the same question in part (a) for 30 unit squares and explain your reasoning.

c) Answer the same question in part (a) for 256 unit squares and explain your reasoning.

d) If you have k shaded unit squares, explain how you can determine whether or not you may use all of these shaded squares to make the border of a larger square. If so, what would be the dimensions of the entire square (with border included)?
Explain your reasoning and sketch a corresponding picture below.

1.1 Exercises

1) Below is a (white) rectangle with length 5 units and width 3 units surrounded by a shaded border. How many shaded border tiles are there?



2) Consider a rectangle with length 10 units and width 4 units surrounded by a shaded border of unit squares like the border in #1. Draw a corresponding picture below and state how many border squares there are.

3) Consider a rectangle with length 100 units and width 37 units surrounded by a shaded border. Draw a corresponding picture and state how many border squares there are. Try to think of a way to do this without having to draw and count every square one by one. Clearly describe how you figured out the number of shaded border squares.

4) Find an algebraic expression that represents the number of squares needed to build a border around a rectangular grid with length L units and width W units.
Draw a picture that represents your thinking and explain how you figured this out.

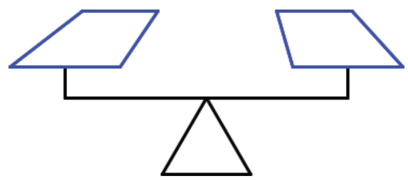
5) Think of a different way (than in #4) to figure out the number of squares needed to build a border around a rectangular grid with length L units and width W units. Write a corresponding algebraic expression and draw a picture. This should look different than your expression for #4 but should be equivalent to it.

6) Show that your algebraic expressions for part #4 and #5 are equivalent using algebraic operations on one or both expressions.

1.2 The Balancing Act

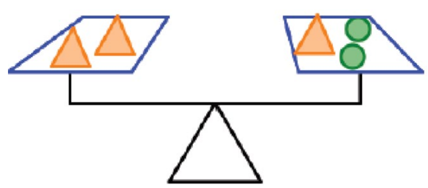
The activities in this section are designed to use pictorial representations to help students understand equality in algebra and begin to work with algebraic equations.

The picture below shows a (very simplistic) two-pan balance scale. Such a scale allows you to *compare* the weight of two objects by placing one object in each pan. If one side is lower than the other, this indicates the lower side holds heavier objects. If the two sides are equally balanced, the objects on each side weigh the same amount.



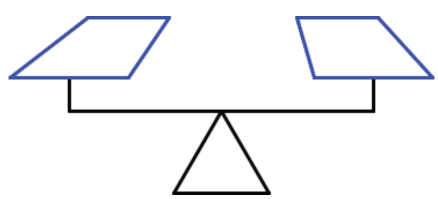
For the following activities, assume each triangle has the same weight, each circle has the same weight, each square has the same weight and each star has the same weight. Also, assume each scale is balanced.

Illustration:Based on the scale picture below we see that 2 triangles weigh the same as one triangle and 2 circles.



If we take a triangle off each side, the scale will still be balanced so we can also say that 1 triangle weighs the same as 2 circles.

Draw shapes on the scale below to represent this situation.

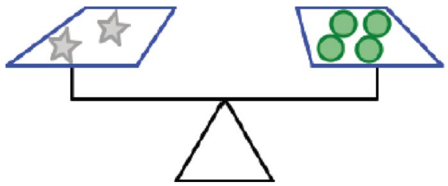


Other ways to describe this relationship between the weight of the circles and triangles are

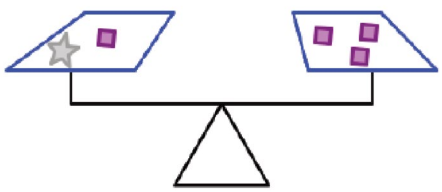
“1 triangle is twice the weight of a circle” or “1 circle is half the weight of a triangle”.

Example 1.2.1

a) Based on the scale picture below, what can you say about the relationship between the weight of a star and the weight of a circle? Explain how you determined your answer.



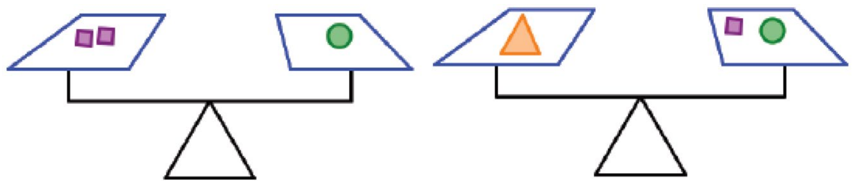
b) Based on the scale picture below, what can you say about the relationship between the weight of a star and the weight of a square? Explain how you determined your answer.



Example 1.2.2

Use Figure 1 to answer the questions that follow.

Figure 1



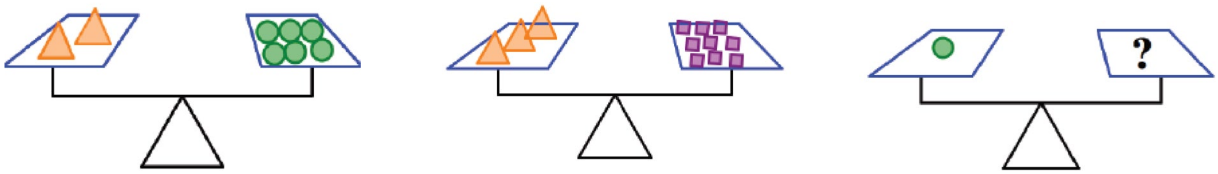
a) Which shape weighs the most? Justify your answer.

b) Which scale is holding the most total weight? How do you know?

Example 1.2.3

Use the first two scale pictures in Figure 2 to gather enough information to determine how many squares go in place of the question mark in the third scale picture. Show your work below.

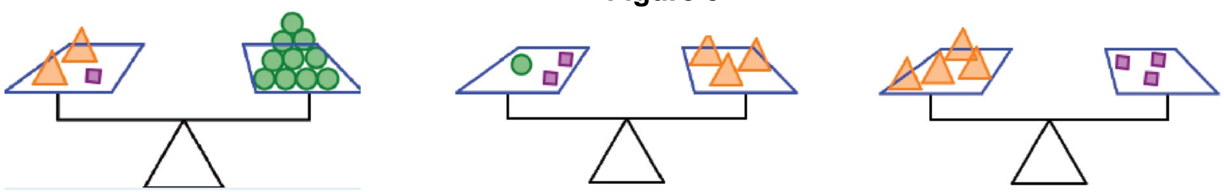
Figure 2

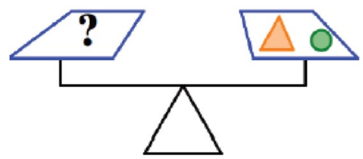


Example 1.2.4

Use the top three scale pictures in Figure 3 to gather information to determine what shapes will balance the scale below. That is, what can we put in place of the question mark that will balance the scale? Try to come up with more than one answer. Explain how you arrived at your answers.

Figure 3

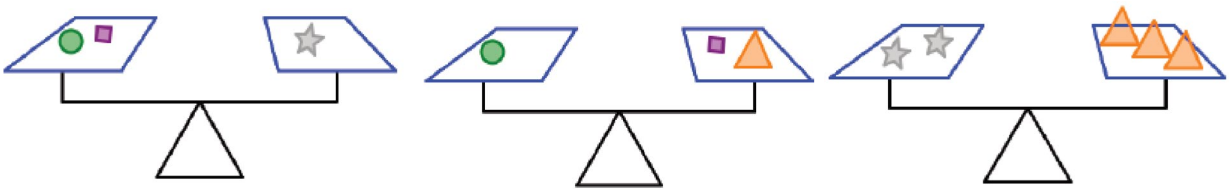




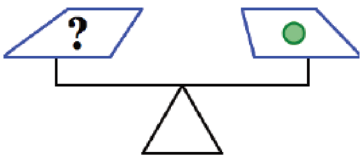
Example 1.2.5

Use the scale pictures in Figure 4 to gather information to determine how many squares go in place of each of the question marks in parts (a), (b) and (c) below.

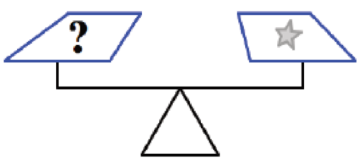
Figure 4



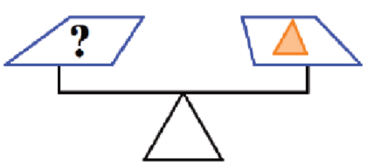
a)



b)



c)



1.2 Exercises

Work through examples 1.2.1 through 1.2.5 again using variables as follows:

Step 1: Write an equation for each scale using the variables below

t = the weight of one triangle
c = the weight of one circle
s = the weight of one star
q = the weight of one square

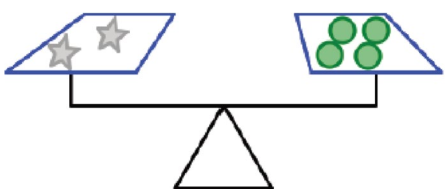
For example, if the (balanced) scale has 4 triangles on one side, and a circle, and a square on the other side, then we write 4t = c + q as the equation.

Step 2: Manipulate these equations to answer the questions. Some manipulations include adding/subtracting equations to get a new equation that is more useful and/or solving for a variable in one equation and plugging it into another equation. Try to avoid using fractions. Show and explain every step. That is, justify where each new equation comes from.

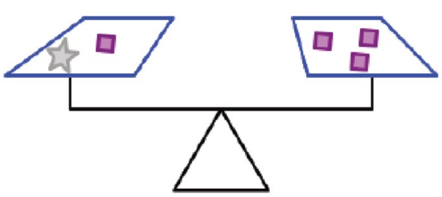
Step 3: State your final answer as a complete sentence (for example: 3 triangles will balance the last scale)

Example 1.2.1 revisited

a) Based on the scale picture, what can you say about the relationship between the weight of a star and the weight of a circle? Explain how you determined your answer .

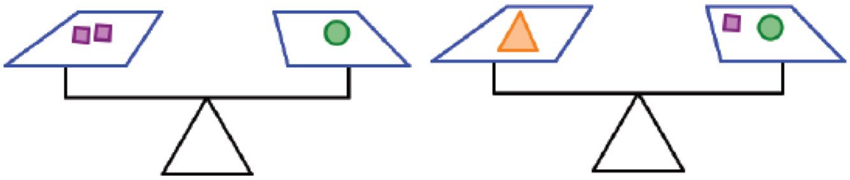


b) Based on the scale picture what can you say about the relationship between the weight of a star and the weight of a square? Explain how you determined your answer.



Example 1.2.2 revisited: Use Figure 1 to answer the questions that follow.

Figure 1

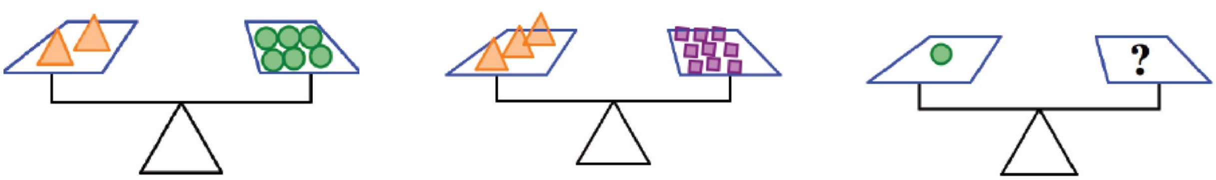


a) Which shape weighs the most? Justify your answer.

b) Which scale is holding the most total weight? How do you know?

Example 1.2.3 revisited: Use the first two scale pictures in the Figure 2 to gather enough information to determine how many squares go in place of the question mark in the third scale picture.

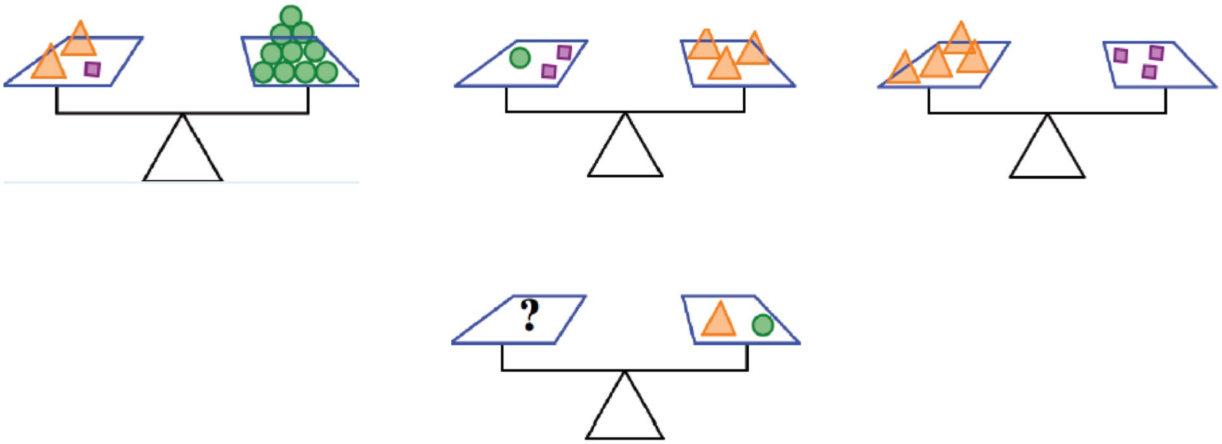
Figure 2



Example 1.2.4 revisited: Use the first three scale pictures in Figure 3 to gather information to determine what shapes will balance the scale below. That is, what can we put in place of the

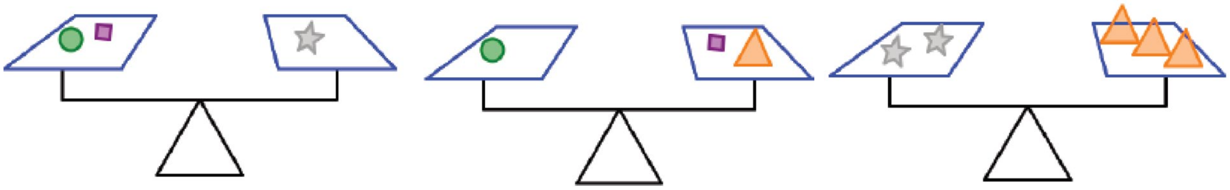
question mark that will balance the scale? Try to come up with more than one answer. Explain how you arrived at your answers.

Figure 3

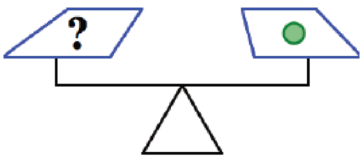


Example 1.2.5 revisited: Use the scale pictures in Figure 4 to gather information to determine how many squares go in place of each of the question marks in parts (a), (b) and (c) below.

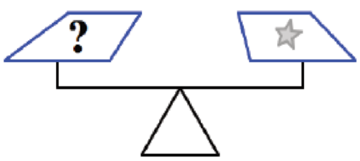
Figure 4



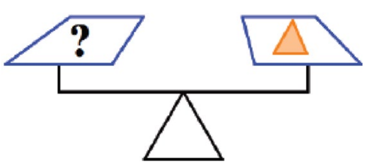
a)



b)



c)



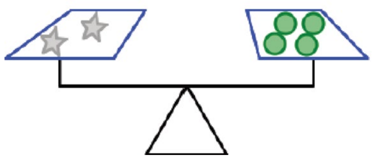
Discussion

Now that you have worked through Examples 1.2.1 through 1.2.5 in two different ways, let us consider the following question:

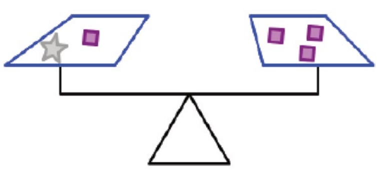
> Which method did you think was easier: using shapes and pictures OR using variables and algebraic expressions? Your responses may vary from example to example so address each one separately. In each case, discuss why you think the chosen method was easier. The examples are copied below for you to annotate.

Example 1.2.1 revisited

a) Based on the scale picture what can you say about the relationship between the weight of a star and the weights of a circle? Explain how you were able to determine the results.



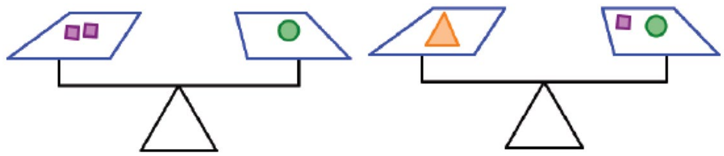
b) Based on the scale picture, what can you say about the relationship between the weight of a star and the weight of a square? Explain how you were able to determine the results.



Example 1.2.2 revisited:

Use Figure 1 to answer the questions that follow.

Figure 1



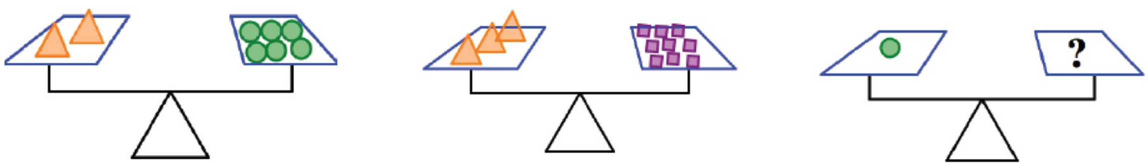
a) Which shape weighs the most? Justify your answer.

b) Which scale is holding the most total weight? How can you prove this?

Example 1.2.3 revisited:

Use the first two scale pictures in the Figure 2 to gather enough information to determine how many squares go in place of the question mark in the third scale picture.

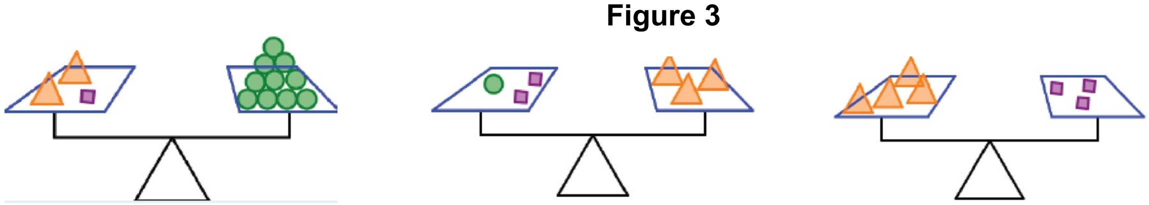
Figure 2

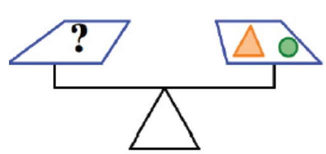


Example 1.2.4 revisited

Use the three scale pictures in Figure 3 to gather information to determine what shapes will balance the scale below. That is, what can we put in place of the question mark that will balance the scale? Try to come up with more than one answer. Explain how you arrived at your answers.

Figure 3

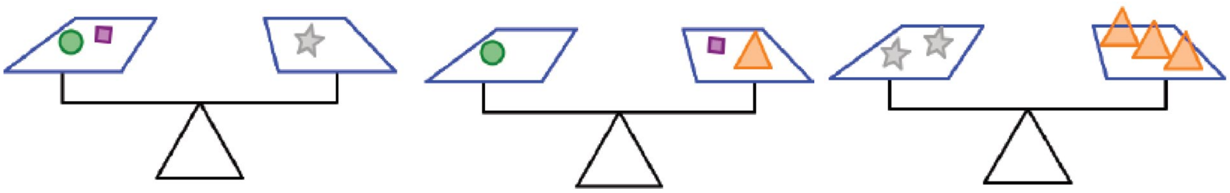




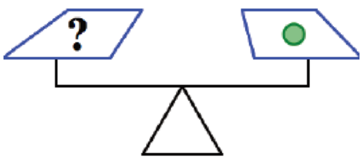
Example 1.2.5 revisited:

Use the scale pictures in Figure 4 to gather information to determine how many squares go in place of each of the question marks in parts (a), (b) and (c) below.

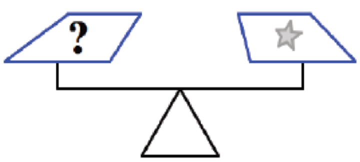
Figure 4



a)



b)



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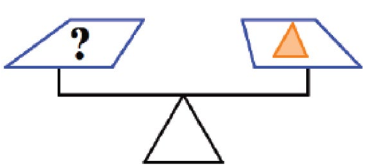


1.2 Activity

Go to
https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pan-Balance----Shapes/
Follow the instructions given and record your work as follows.

1) Record your table results below.

2) Use your table results to try to determine the weight of each shape. Clearly show your work below.

3) Click on “Guess Weights” to see if you are correct. If your answers are not all correct, go back to do more weighings and/or check your work from step 2 and try to guess again. If you are having trouble, try the strategy in the Exploration to help.

Later in this chapter, we will explore a different type of scale balancing problem that relies more on the use of variables and equations; first, we address the concepts and use of notation and variables in algebra.

1.3 Understanding Equality in Algebra

The notion of equality is fundamental in mathematics and especially in algebra and algebraic thinking. The symbol "="' expresses a *relationship*. It is *not* an operation in the way that + and are × operations. It should not be read left-to-right, and it definitely does not mean "… and the answer is …".

For your work to be clear and easily understood by others, it is essential that you use the symbol = appropriately. For your future students to understand the meaning of the = symbol and use it correctly, it is essential that you are clear and precise in your use of it.

Example 1.3.1

Consider the following problem.

> *Akira went to visit his grandmother, and she gave him $1.50 to buy a treat. He went to the store and bought a book for $3.20. After that, he had $2.30 left. How much money did Akira have before he visited his grandmother?*

Kim solved the problem as follows:

$$2.30 + 3.20 = 5.50 - 1.50 = 4. \text{ \textit{So the answer is 4}}$$

a) What do you think about Kim's solution? Did she get the correct answer?

b) Is her solution clear? How could it be better?

c) Rewrite Kim's solution making correct use of the equal sign. Show your work below.

An **equation** is a statement asserting that two expressions have the same numerical value (represent the same real number). Like all statements, equations may be true or false. All equations and inequalities fall into one of three categories:

Always true: for example, $2x - 3 = 1 + x - 4 + x$
Never true: for example, $2x = 2x + 5$
Sometimes true: for example, $3x - 5 = 10$

Example 1.3.2

Examine the following equations. Decide whether the statement is always true, never true or sometimes true. Record your answers below and explain how you know.

a) $5 + 3 = 8$	b) $5k = 5k + 1$
c) $5 + 3 = y$	d) $\frac{2}{3} + \frac{1}{2} = \frac{3}{5}$
e) $n + 3 = m$	f) $\frac{a}{5} = \frac{5}{a}$
g) $3x = 2x + x$	

Example 1.3.3

Consider the equation 18 - 7 = ____

a) What can you put in the blank that will make the equation always true?

b) What can you put in the blank that makes the equation always false?

c) What can you put in the blank that makes the equation sometimes true and sometimes false?

A **solution** to an equation is a value for the variable or variables that makes the equation true.

Illustration:

a) The solution to $2x - 3 = 1 + x - 4 + x$ is any real number x
b) 2x = 2x + 5 does not have any solutions
c) The solution to 3x - 5 = 10 is x = 5.

In general English, a ‘solution’ is a way to fix a problem. And even in math, if you are not talking about an equation, the word ‘solution’ is often used to refer to the ‘answer’ or more accurately ‘a method for finding the answer’ to a problem. But, in the context of equations, the ‘solution’ to an equation does not mean "the answer" or "a method for finding an answer,"but rather it means "the values for the variables that make the equation true."

Example 1.3.4

Consider the following problem: 6 + 4 = ____ + 3

a) Jose thinks that 10 goes in the blank. Why might he say this, and what would you say to him as his teacher?

b) Mari argues that 7 goes in the blank because 6 and 4 is 10. And you have to add 7 to three to get 10. Peter argues that 7 goes in the blank because 3 is one less than 4 and so you have to fill in the blank with one more than 6. How are Mari and Peter's arguments different? As the teacher of these children, what do these two responses tell you about what they know? Record your answers below.

As a teacher you will need to help your students understand that '=' does not simply mean 'write the answer,' and instead help them to see the equation as a balance.

Example 1.3.5

How might you use a balanced scale to demonstrate for your class a solution to the problem: 6 + 4 = ____ + 3. Draw a corresponding picture below.

Your use of the '=' sign is important; your students will be watching and learning from the way you write and talk about mathematics. Remind them that an equation is like a balanced scale: whatever you put on or take away from one side, you must do to the other side to maintain the balance.

Give your students experiences that will help them think this way. Write equations (with blanks for them to fill in) in a variety of ways.

Illustration

Different ways to ask fill in the blank questions with equalities

a) ___ = 6 + 4 may be rewritten as 10 = ___ + 6 or 10 – ___ = 4

b) ___ x 3 = 24 may be rewritten as 24 ÷ ___ = 8

Example 1.3.6

Fill in the blank for each equation and then rewrite the problem with the blank in a different place.

a) 13 – 7 = ___

b) 7 x 4 = ___

c) 15 ÷ 3 = ___

d) 12 + 5 = ___

1.4 The Use of Variables in Algebra

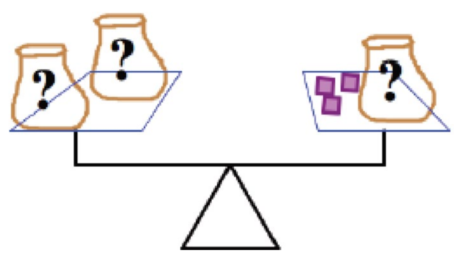
Pre-Class Work

Go to the following link https://www.geogebra.org/m/fjekqnuh

1) Use the sliders to balance the scale according to the given equation
2) Click on "solve" in the lower right corner and use the slider that appears to view the steps. Review the steps carefully (more than once if needed) to make sure you understand the connection between the scale and the algebra.
3) Click on "new problem" and repeat the process.
4) Go through at least 3 problems like this.
5) Draw a scale picture that represents the equation $4x + 5 = 21$. Solve this equation algebraically and for each step, draw a corresponding scale picture that represents the equality. Be prepared to share in class.

Illustration

The figure below shows bags containing bags and single blocks on a scale. The bags are marked with a "?" because they contain some unknown number of blocks. Each bag contains the same number of blocks and the scale is balanced.



In order to determine how many blocks are in each bag we can take one bag from each side of the scale and see that one bag weighs the same as 3 blocks and hence each bag contains 3 blocks.

Alternatively, we may use a variable in an equation as follows:

Let x = the number of blocks in each bag.
Create an equation involving x based on the fact that the scale is balanced.

$2x = x + 3$

Solve for x.

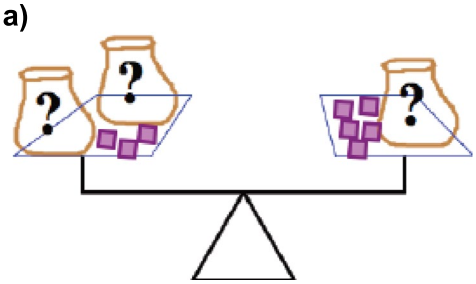
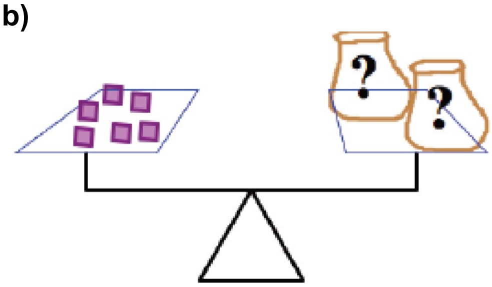
$2x = x + 3$ (subtract x from both sides)
$x = 3$

Notice how subtracting x from both sides corresponds to removing one bag from each side of the scale.

Example 1.4.1

For parts (a) and (b), determine the number of blocks in each bag.
Explain your answer in two ways:

(1) without using a variable or equation
(2) by forming an equation with a variable and solving it.

a) 	**b)** 
(1) without using a variable or equation	(1) without using a variable or equation
(2) by forming an equation with a variable and solving it.	(2) by forming an equation with a variable and solving it.

Example 1.4.2

Draw a scale figure similar to those in Example 1.4.1, that represents the equation $5x + 2 = 3x + 8$ where x is the number of blocks in each bag. Then, solve the equation. For each step, draw a corresponding scale picture and explain what scale action was taken to get from the previous step to the current step.

Example 1.4.3

Draw a (bag/block) scale figure that represents each of the following equations and state what the variable represents. Then, solve the equation. Explain the implications of your results in terms of the scale figure, as if you were talking to an elementary school student.

a) $4p + 7 = 4p + 7$	b) $3y + 7 = 3y + 2$

Example 1.4.4

Create a (bag/block) scale figure problem where the answer is not a whole number. Draw a corresponding picture below. Solve it and explain your answer in terms of bags and blocks.

Different Types of Variables

The type of variable that we are dealing with depends on how it is used (in an equation).

Type of variable	**Illustration**
Variable as a single unknown quantity (that we can find)	In the equation $2y = 3 - 4y$ y is a variable that is a single unknown that we can solve for and get a single value
Variables as *quantities that relate* to each other (in an equation with two or more variables)	In the equation $L \cdot W = 24$, *L* and *W* are related to each other in a way that the value of one determines/depends on the value of the other.
Variables that can be *any number* (in an equation that is always true, as in a rule or property)	In the equation $a(b + c) = ab + ac$ a, b, and c may be any number since the equation is always true (distributive property)

Variable ***type*** is not to be confused with ***what a variable represents*** in a given scenario. For example in the equation $L \cdot W = 24$, *L* and *W* are *quantities that relate*, referring to the type of variable they are as described in the table above.

But when describing what each variable represents in the scenario of rectangle area equalling 24, *L* represents the length of the rectangle and *W* represents the width of the rectangle.

Example 1.4.5

Let *h* represent the number of hours and *d* represent the number of days.

a) Which of the following equations correctly shows a relationship between the two variables? Explain your answer and give an example using specific numbers.

$$h = 24d \quad \text{or} \quad d = 24h$$

b) What types of variables are h and d, based on the table above?

Illustration

a) Consider the following scenario: Drena has double $300 more than Jasper and together they have $900.

   This scenario may be represented by the following equation: $2(x+300)+x=900$

   Variable type of x: single unknown

   What does x represent? The amount of money that Jasper has

b) Assume the equation $25x=2y$ represents that there are two teachers for each class of 25 students.

   Variable type of x and y: quantities that relate

   What do x and y represent?
   x represents the number of teachers and y represents the number of students.

c) Consider the following equation: $(a \cdot b)^x = a^x \cdot b^x$

   Variable type of a, b and x: Any number

   What do a, b and x represent? Any number

The only situation in which we get the same answer for variable type and what the variable represents is when we have an equation that is always true no matter what values we plug in for them as in part (c) of the illustration above.

Example 1.4.6

For each of the following scenarios and corresponding equations, state what each variable represents in terms of the given situation and state what type of variable it is, based on the table above (*single unknown, quantities that relate, any number*).

a) Jazz has 12 coins in dimes and quarters, for a total of $1.95 (one dollar and 95 cents) in change: $25x + 10(12 - x) = 195$

Variable type of x:

What does x represent?

b) The square root of a product is the product of the square roots: $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

Variable type of a and b:

What do a and b represent?

c) To make an apple pie you need 3 cups of sugar for every 10 apples: $y = \frac{10}{3}x$. This one is a bit tricky so it might be helpful to plug in some numbers.

Variable type of x and y:

What do x and y represent?

Example 1.4.7

Consider the following scenario. Three chickens weigh the same as a monkey. Two monkeys and a chicken weigh the same as a sheep. Let C represent the weight of one chicken, M represent the weight of one monkey and S represent the weight of one sheep.

a) Write two equations that represent the relationships in the problem.

b) Determine how many chickens weigh the same as one sheep.

c) What types of variables are *C*, *M* and *S*?

Example 1.4.8

Write word problems that would be appropriate for each of the following equations. Try to incorporate some cultural diversity into your scenarios that might resonate with a diverse student population.

a) $x - 11 = 45$

b) $3x = 42$

c) $3x + 12 = 4x + 10$

1.4 Exercises

1) Here is an expression for the cost of 7 shirts and 3 hats: $C = 7x + 3y$.

    a) What precisely does the x represent?

    b) What precisely does the y represent?

    c) What precisely does the *C* represent?

2) Here is an expression for the cost in (dollars) of purchasing n pineapples and m papayas: $5n + 2m$.

    a) What is the meaning of the number 5 in the expression?

    b) Which costs more: a pineapple, or a papaya? How can you tell?

3) In an inner city school, there are 20 times as many students as teachers. Let S represent the number of students and T represent the number of teachers.

    a) Explain why the equation 20S = T does NOT represent the relationship between the number of teachers and the number of students.

    b) What is the correct equation for this relationship? Explain why.

4) Let *P* be the initial population of a town. After 10% of the town leaves, another 1400 people move into the town. The new population is 5000. Write an equation, involving *P*, that corresponds to this situation.

5) Write a word problem that you could use the following equations to solve. You do not have to solve it.

a) $8 + x = 20$

b) $3 = \frac{42}{x}$

6) To make orange paint, you need to mix yellow and red paint together. For a certain shade of orange paint, you need to use $2\frac{1}{2}$ times as much yellow paint as red paint. Write an equation that relates quantities in this situation. Be sure to define your variables clearly and correctly.

7) At the cafeteria, you can get a lunch special that offers a drink and salad. The drink costs \$0.75 and a salad costs \$0.25 for every ounce. Define two corresponding variables and write an equation to show how the total cost of a lunch special is related to the number of ounces of salad one gets.

8) Max was asked to write an equation corresponding to the following situation:
There are 2 times as many pencils in Antrice's pencil box as in Keertana's pencil box.
Max responded with the following:

A = Antrice K = Keertana
2A = K

Discuss Max's work, describing and correcting any errors he has made. Explain Max's errors and how you arrived at your answer.

9) There are *F* cups of flour in a bag. First, $\frac{3}{8}$ of the flour in the bag was used. Then $\frac{1}{5}$ of the remaining flour was used. At that point, there were ten cups of flour in the bag. Write an equation, involving *F*, that corresponds to this situation. Explain your work.

10) Describe your own situation in which two quantities vary together. Define two variables and write an equation to show how they are related. Use a table and/or picture (e.g. bar diagram) to help show why your equation is valid.

Chapter 1 Activity

Solving Algebraic Equations-Making Connections to Balance Act

This activity will help us further our understanding of the role/meaning of equality sign by utilizing the balance scale with expressions to solve an algebraic equation and revisit the meaning of “solution of an equation.”

1. Go to the following link: https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pan-Balance----Expressions/
   Follow the instructions to get familiar with the app, then complete the exploration. Provide your answers to the exploration questions #2-6 below:

   2)

   3)

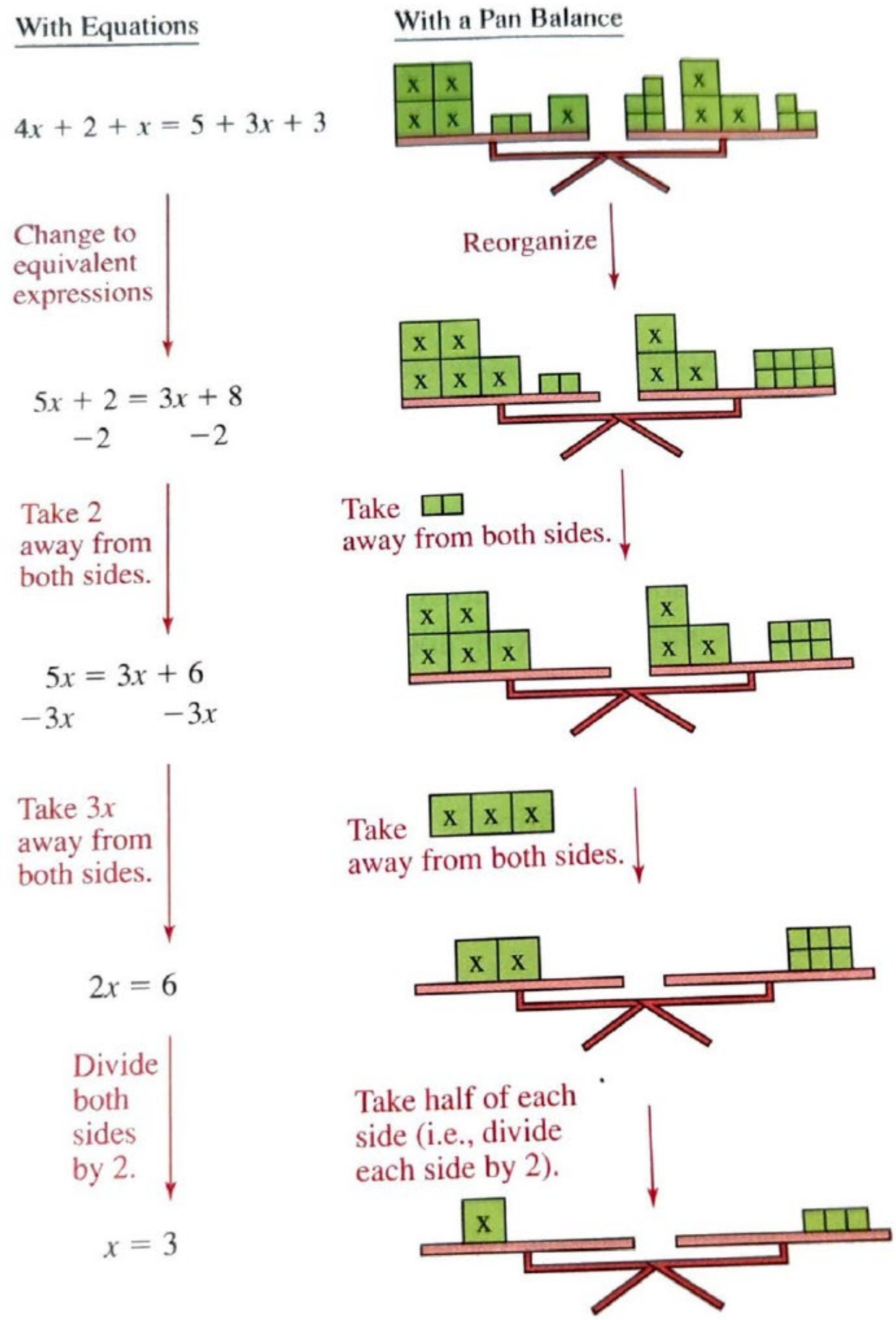
   4)

   5)

   6)

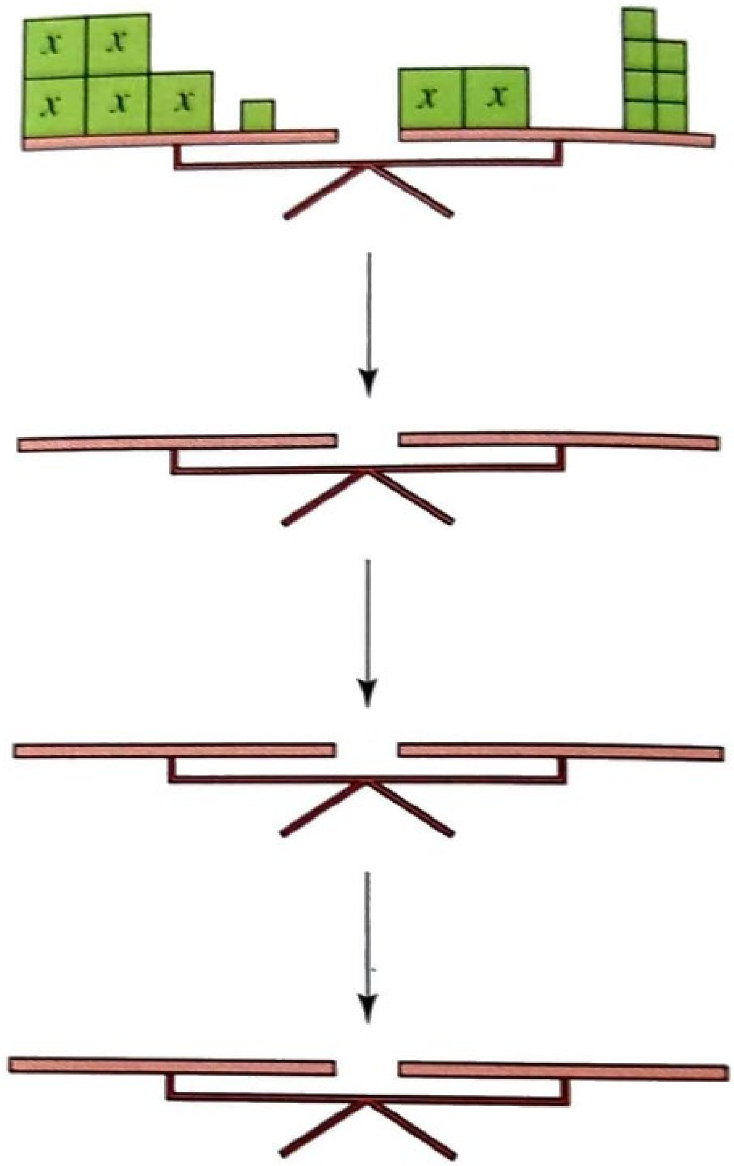
Try another equation and solve it using this app. Write your equation and solution below and explain how you used the app to solve it.

2. Review the following model (from *Mathematics for Elementary Teachers* by Sybilla Beckman), showing how to solve equations algebraically and using a pan balance. Consider the following questions: (a) What did you learn about solving an equation? (b) What similarities and differences do you see with this activity and the previous scale activities? (c) How might an elementary school teacher use the Pan Balance to explain the process of solving an equation algebraically?



For extra explanation and examples watch the following videos:
https://www.youtube.com/watch?v=CKcbgB8TLEU and
https://www.youtube.com/watch?v=_JLr3fkgUhU

3. Solve $5x + 1 = 2x + 7$ in two ways, (1) with equations and (2) with pictures of a pan balance. Relate the two methods with common explanations as done in #2.

With equations	With a pan balance
$5x + 1 = 2x + 7$	

4. Write an equation involving variable x. Solve your equation in two ways: (1) with equations and (2) with the pictures of a pan balance. Relate the two methods with common explanations as done in #2.

Chapter 1: Extra Exercises

With more advanced equations (first review corresponding algebraic properties as needed)

1) Maria was given the following problem to solve: $\frac{4x}{x+2} = 3 - \frac{8}{x+2}$

   Her work is below. Explain what she did for each step and why her final answer is not valid.

   $4x = 3(x+2) - 8$
   $4x = 3x + 6 - 8$
   $4x = 3x - 2$

   The solution is x = -2

2) Yihang was given the following problem to solve: $\frac{1}{3}x - 2 = \sqrt{4-x}$

   His work is below. Explain what he did for each step and why his final answer is not valid.

   $\left(\frac{1}{3}x - 2\right)^2 = 4 - x$

   $\frac{1}{9}x^2 - \frac{4}{3}x + 4 = 4 - x$

   $x^2 - 12x + 36 = 36 - 9x$

   $x^2 - 3x = 0$

   $x(x-3) = 0$

   The solutions are x = 0 and x = 3

3) For parts (a) - (c) an equation is given along with three attempts by various students to write an equivalent equation. For each student in each part, determine if their equation is equivalent to the original equation or not. Explain why or why not.

a) Original equation: $(x+3)^2 = 4x^2 - 36$

Alice: $x + 3 = 2x - 6$

Bubba: $x^2 + 9 = 4x^2 - 36$

Dalia: $x^2 = 4x^2 - 39$

b) Original equation: $x + \sqrt{x^2 + 9} = 4$

Bubba: $x^2 + (x^2 + 9) = 16$

Dalia: $x + (x + 3) = 4$

c) Original equation: $\frac{3x}{x+4} = 2x - 5$

Bubba: $\frac{3}{4} = 2x - 5$

Dalia: $3x = 2x - 5(x + 4)$

Chapter 1 wrap up

The main goal of this chapter is to show how important algebraic thinking is at the elementary level and how it “grows up” with the use of variables in the later grades. Hopefully it helped you understand and appreciate the power of algebra as something you can pass on to your future students.

Algebra is the foundation of mathematical modeling. In subsequent chapters we will explore specific types of models in many different forms: pictures, graphs, tables, equations, and more. In particular, the next chapter expands on forming models via pattern recognition, similar to some of the examples in the beginning of this chapter.