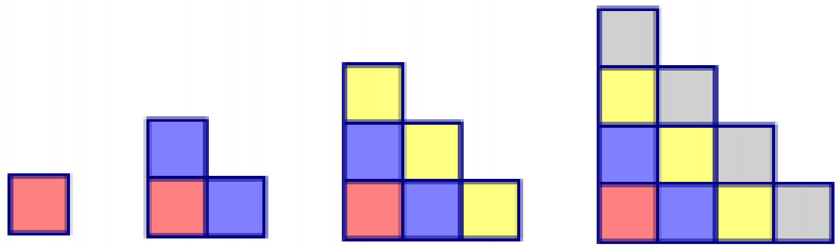
Chapter 2: PATTERNS



"Patterns are everywhere. Children who are encouraged to look for patterns and to express them mathematically begin to understand how mathematics applies to the world in which they Live." --(National Council of Teachers of Mathematics [NCTM], 1989, p. 260)

In this chapter…

See the corresponding standards below from The **National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics**

In prekindergarten through grade 2 all students should…
• recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another
• analyze how both repeating and growing patterns are generated
• describe quantitative change, such as a student's growing two inches in one year.

In grades 3–5 all students should
• describe, extend, and make generalizations about geometric and numeric patterns
• represent and analyze patterns and functions, using words, tables, and graphs
• investigate how a change in one variable relates to a change in a second variable.

These skills build up to corresponding middle school standards below

In grades 6-8 all students should
• represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
• relate and compare different forms of representation for a relationship;

In the **Massachusetts Mathematics Curriculum Framework**…

Grade 3 - 3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operation

Grade 4 - 4.OA.C Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Grade 7 - 7.EE.B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    c. Extend analysis of patterns to include analyzing, extending, and determining an expression for simple arithmetic and geometric sequences (e.g., compounding, increasing area), using tables, graphs, words, and expressions.

Patterns may be observed in real-world situations, geometric figures, numbers, symbols, and relationships among quantities. Patterns are an excellent way to introduce students to making predictions and conjectures. Recognizing patterns is often considered the foundation for algebraic thinking. In particular, we can use algebra to simplify the pattern and develop an expression for the general case. Using symbols to describe the general rule of a pattern is central to algebraic thinking.

Teaching of patterns begins in the elementary classroom with repeating patterns followed by growing patterns. The study of patterns prepares students to work with functional relationships which are addressed later in this text.

2.1 Repeating Patterns

Repeating patterns are often introduced to children using colored blocks, chips, or some other object. It is important to expose students to different objects in repeating patterns so they are able to translate between different representations of the same type of pattern. This helps them understand patterns much like using different variables helps one understand algebra. Eventually they learn to write out patterns on paper using letters (for example: ABABAB…).

Watch this video as a sample of how the study of patterns begins in elementary school (~ 2 minutes):

https://www.youtube.com/watch?v=1WJcYwSu0-U

Watch the following video for a more general understanding of patterns and type of patterns (~4 minutes):

https://youngmathematicians.edc.org/math-topic/patterns-and-algebra/

Example 2.1.1

Assume the colors represent colored blocks.

a) Consider the following pattern (showing two repetitions):

red, red, green, red, red, green,...

Suppose this pattern continued. What color will the 25th block be? Explain how you got your answer.

b) Consider the following pattern (showing two repetitions):

red, green, blue, red, green, blue,...

Suppose this pattern continued to 25 blocks. How many red blocks would there be in the pattern? Explain how you got your answer.

c) Consider the following pattern (showing two repetitions):

yellow, green, green, yellow, green, green,...

Suppose this pattern continued until there were 9 yellow blocks total and then stopped. How many green blocks would be in the pattern? Explain how you got your answer.

How can we figure out answers to Example 2.1.1 without writing out all the blocks? Discuss.

Observe the quotient and remainder are important concepts in answering repeating pattern questions, especially when the numbers get too large to list out the objects.

The **division algorithm** for whole numbers states that for any **dividend** $\boldsymbol{n}$ and **divisor** $\boldsymbol{d > 0}$ there is a unique **quotient** $\boldsymbol{q}$ and unique **remainder** $\boldsymbol{r}$ with $0 \leq r < d$, such that $\boldsymbol{n = q \times d + r}$

Illustration (division algorithm):

Consider $265 \div 3$. Our result is 88 ⅓ or 88 with remainder 1 so the following equality holds $265 = 88 \times 3 + 1$. In this problem n = 256 is the *dividend*, d = 3 is the *divisor*, q = 88 is the *quotient* and r = 1 is the *remainder*.

Recall Example 2.1.1

a) Consider the following pattern: red, red, green, red, red, green,...
Suppose this pattern continued. What color will the 25th block be?

Using division to solve this problem, determine the following values and explain which one gives you your answer and why. Write the equation $n = q \times d + r$ with your numbers filled in to check equality.

Dividend (n):

Divisor (d):

Quotient (q):

Remainder (r):

$n = q \times d + r$:

b) Consider the following pattern: red, green, blue, red, green, blue,...
Suppose this pattern continued to 25 blocks. How many red blocks would there be in the pattern?

Using division to solve this problem, determine the following values and explain which one gives you your answer and why. Write the equation $n = q \times d + r$ with your numbers filled in to check equality.

Dividend (n):

Divisor (d):

Quotient (q):

Remainder (r):

$n = q \times d + r$:

Example 2.1.2

1) How does the quotient relate to repeating pattern problems (in general). That is, what does the quotient represent in repeating pattern problems?

2) How does the remainder relate to repeating pattern problems (in general). That is, what does the remainder represent in repeating pattern problems?

Example 2.1.3

Create a repeating pattern and write three questions about it that you could ask children to promote their algebraic reasoning. Make the numbers big enough to motivate the use of the division algorithm as opposed to writing out all the blocks.

Then partner with someone in the class and try to answer each other's questions.

Pattern:

Question 1:

Question 2:

Question 3:

Example 2.1.4

Beginning with 2, the digits in the ones place of even numbers are 2, 4, 6, 8 or 0. What digit is in the ones' place of the 358th even number? Explain how you got your answer.

Example 2.1.5

A ribbon design consists of an elephant, a lion, a giraffe and a zebra in that order. Each figure on the ribbon is 5 centimeters wide. If you cut a 1468 cm long piece of the ribbon that starts with an elephant, what will be the last figure on your piece of ribbon? Explain how you got your answer.

2.1 Exercises

1) Consider the following block pattern (showing two repetitions):
   yellow, green, green, red, yellow, green, green, red,...
   Answer the following questions, assuming the pattern continues.
   a) What color will the 291th block be? Explain how you arrived at your answer.

   b) How many of each color block will there be when there are 258 blocks altogether? Explain how you arrived at your answer.

   c) How many green blocks will you have used when you place the 30th yellow block in the pattern? Explain how you arrived at your answer.

2) Consider the following block pattern (showing two repetitions):
   green, blue, green, green, red, green, blue, green,green, red,....
   a) What color will the 391st block be? Explain how you arrived at your answer.

   b) If you have extended the pattern until you used exactly 56 green blocks, how many blocks total are there? Explain how you arrived at your answer.

3) You get a 90-day temporary license plate for your new car. If you got the license on a Monday, on what day will it expire?

2.2 Growing Patterns

Pre-Class Work

Go to https://www.geogebra.org/m/snwkdwxj

1) For the first problem, follow the instructions in building the 4th figure and check it. Sketch the first 4 figures below.

2) Enter an equation as prompted, record your equation below and state exactly what each variable represents.

3) Explain how you determined your equation, even if it is not correct.

4) If your equation is not correct, keep trying until you either get it correct or you are given a “Possible Solution”. Record the result (your correct solution or the “possible solution”) below and explain why it is correct.

5) Try another problem by clicking on “New Problem”

In this section we look at different types of patterns known as *growing patterns* where each subsequent figure grows in a distinct way. We start with patterns of growing tile figures.

Example 2.2.1

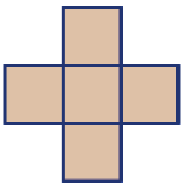


Figure 1

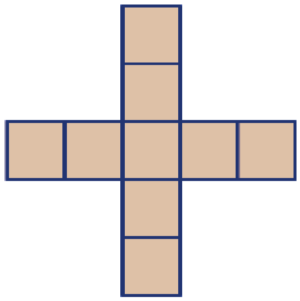


Figure 2

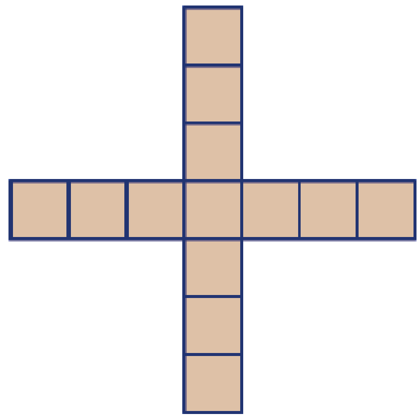


Figure 3

a) Describe in words how you see the pattern growing. Focus on what stays the same and what changes from one figure to the next.

b) Calculate the number of tiles you would need to build each figure up to figure 10 and record your answers in the table below.

Figure	1	2	3	4	5	6	7	8	9	10
# of tiles										

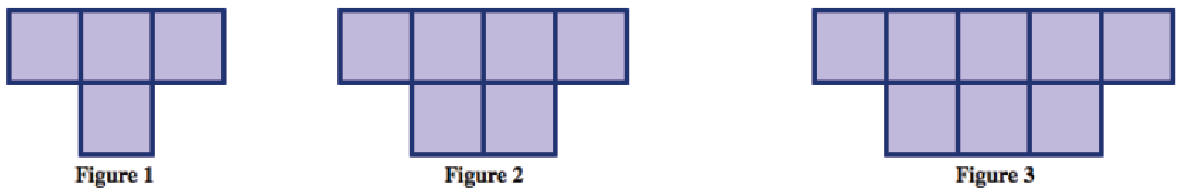
c) Calculate the number of tiles you would need to build the 100th figure in the pattern.

d) Describe how you can figure out the number of tiles in any figure in the pattern. In particular, come up with a formula for the number of tiles in the *n*th figure. Justify your answer based on how the pattern grows.

e) Could you make one of the figures in the pattern using exactly 25 tiles? If yes, which figure? If no, why not? Justify your answer.

f) Could you make one of the figures in the pattern using exactly 100 tiles? If yes, which figure? If no, why not? Justify your answer.

Example 2.2.2



a) Describe in words how you see the pattern growing. Focus on what stays the same and what changes from one figure to the next.

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Figure	1	2	3	4	5	6	7	8	9	10
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c) Calculate the number of tiles you would need to build the 100th figure in the pattern.

d) Describe how you can figure out the number of tiles in any figure in the pattern. In particular, come up with a formula for the number of tiles in the nth figure. Justify your answer based on how the pattern grows.

e) Could you make one of the figures in the pattern using exactly 25 tiles? If yes, which figure? If no, why not? Justify your answer.

f) Could you make one of the figures in the pattern using exactly 100 tiles? If yes, which figure? If no, why not? Justify your answer.

Example 2.2.3

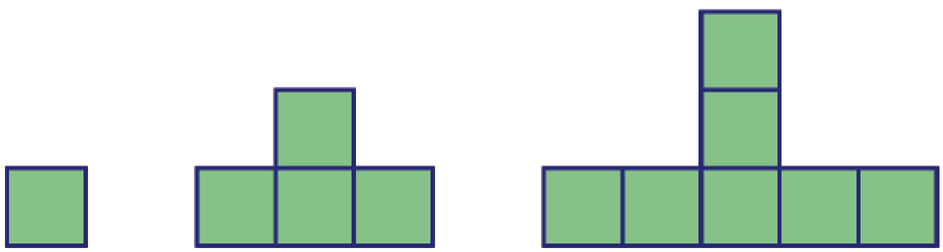


Figure 1 Figure 2 Figure 3

a) Describe in words how you see the pattern growing.Focus on what stays the same and what changes from one figure to the next.

b) Calculate the number of tiles you would need to build each figure up to figure 10 and record your answers in the table below.

Figure	1	2	3	4	5	6	7	8	9	10
# of tiles										

c) Calculate the number of tiles you would need to build the 100th figure in the pattern.

d) Describe how you can figure out the number of tiles in any figure in the pattern. In particular, come up with a formula for the number of tiles in the *n*th figure. Justify your answer based on how the pattern grows.

e) Could you make one of the figures in the pattern using exactly 25 tiles? If yes, which figure? If no, why not? Justify your answer.

f) Could you make one of the figures in the pattern using exactly 100 tiles? If yes, which figure? If no, why not? Justify your answer.

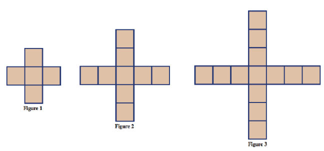
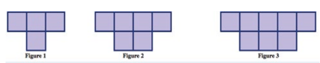
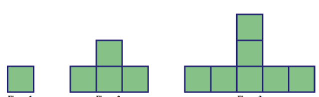
There are two main ways to describe growing patterns as follows:

Way 1: Using a **recursive formula** which determines the number for the next figure using the number for the previous figure. Recursive formulas have two components:

1) The number for the first figure (first figure number = ?)
2) A rule that describes the number for the nth figure by adding something to the number for the previous figure (nth figure number = ? in terms of figure (n-1) number)

Way 2: Using an **explicit formula** which gives a rule for the nth figure without using previous figures (only uses the figure number n). For explicit formulas we determine a rule for the pattern and express it as a general formula for the desired number in figure n.

For Example 2.2.1, check the recursive and explicit formulas (in the table below) for several n values. Then complete the recursive formulas for Examples 2.2.2 and 2.2.3 (in the table below) by replacing the question marks with appropriate numbers.

	Pattern	Recursive Formula	Explicit formula
Example 2.2.1		1) Figure 1 (area) = **5** 2) Fig. n (area) = Fig.(n-1) area + **3** For n = 2,3,4,...	4n +1 for n = 1, 2, 3, …
Example 2.2.2		1) Figure 1 (area) = **?** 2) Fig. n (area) = Fig.(n-1) area + **?** For n = 2,3,4,...	2n + 2 for n = 1, 2, 3, …
Example 2.2.3		1) Figure 1 (area) = **?** 2) Fig. n (area) = Fig.(n-1) area + **?** For n = 2,3,4,...	3n - 2 for n = 1, 2, 3, …

Think - Pair - Share

Think about how the recursive and explicit formulas for each pattern above are related.
Pair up with someone in the class to share each other's thoughts on the matter.
Describe in general, how the recursive and explicit formulas are related so that your explanation could be used for any of the three examples. Then check your reasoning with each example.
Record your explanation and work below.

Example 2.2.4

Assume each tile in the patterns from examples 2.2.1 through 2.2.3 is a 1 cm by 1 cm square. Determine recursive and explicit formulas for the **perimeter** of the *n*th figure in each pattern. Explain how you arrived at your answers.

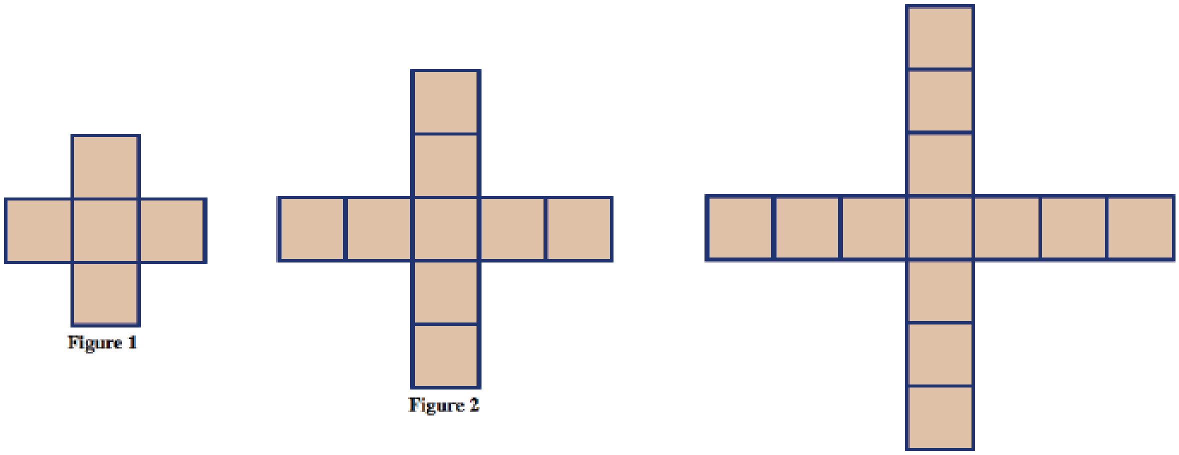


Figure 1 Figure 2 Figure 3

Recursive Formula for perimeter:

Explicit Formula for perimeter:

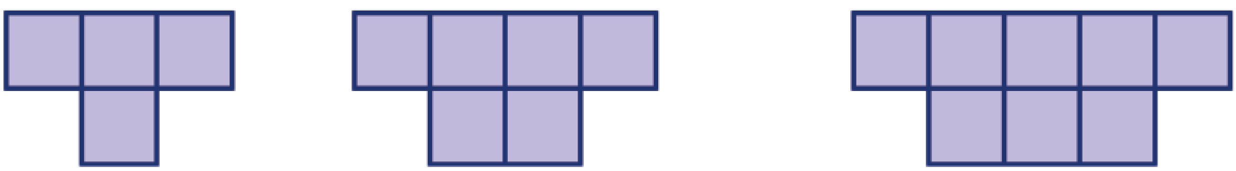


Figure 1 Figure 2 Figure 3

Recursive Formula for perimeter:

Explicit Formula for perimeter:

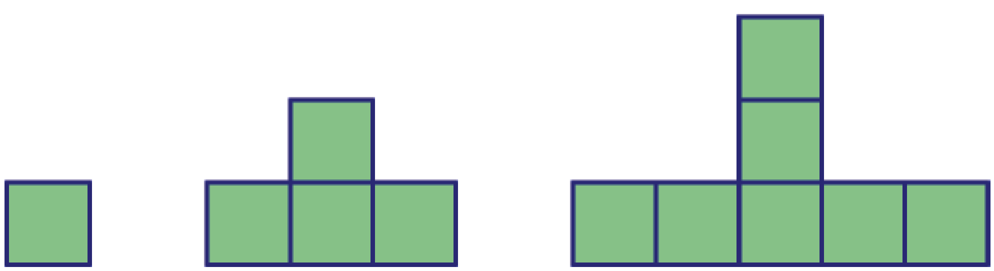


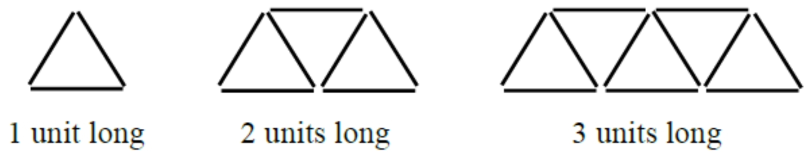
Figure 1 Figure 2 Figure 3

Recursive Formula for perimeter:

Explicit Formula for perimeter:

Example 2.2.5

A common way of constructing bridge trusses (frames made from triangles) is shown below. Observe the truss that is 1 unit long is constructed from 3 unit-long beams, the truss that is 2 units long is constructed from 7 unit-long beams and the truss that is 3 units long is constructed from 11 unit-long beams.



a) Finishing filling in the table below

Truss length	1	2	3	4	5	6	7	8	9	10
# of unit beams needed	3	7	11							

b) How many beams are required to make a truss that is 50 units long? Explain how you got your answer.

c) Write a recursive formula for the number of beams needed to make a truss n units long. Explain how you got your answer.

d) Write an explicit formula for the number of beams needed to make a truss that is n units long. Explain how you got your answer.

e) Share your formula and explanation with a classmate. If your formulas and explanations are the same, write another explicit formula for the number of beams required to make a truss n units long, counting the beams in a different way. Explain how you got your answer. Then prove the two expressions are equivalent using algebraic operations.

Example 2.2.6

Consider the growing pattern below. Answer the questions that follow. Explain and/or show how you got your answers.

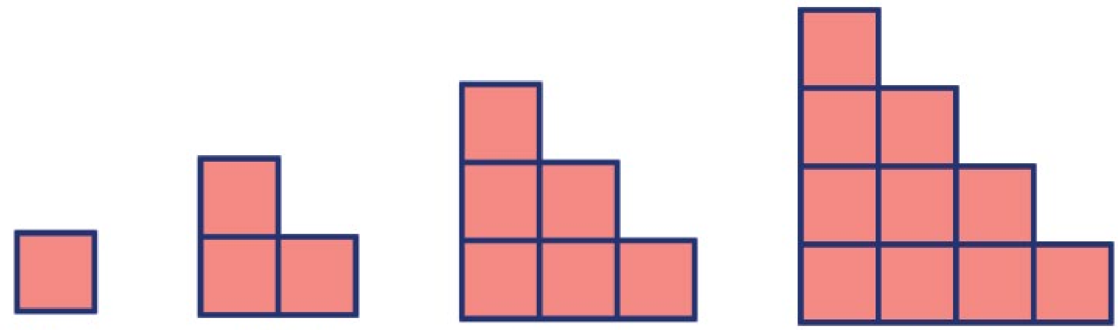


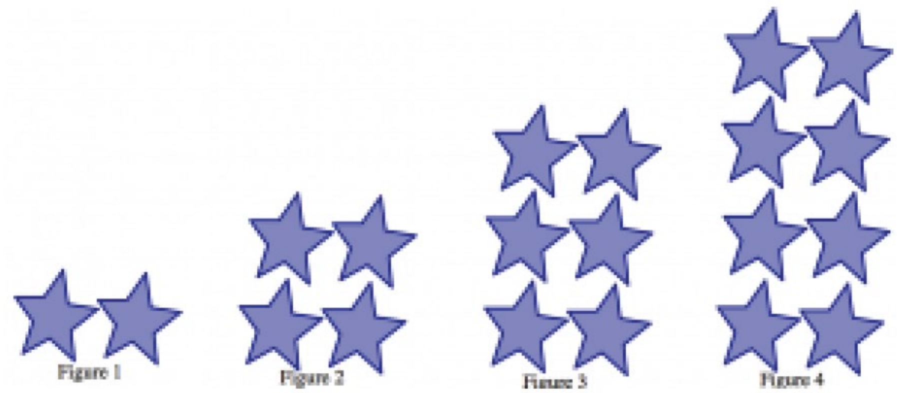
Figure 1 Figure 2 Figure 3 Figure 4

a) Write a recursive formula for the number of tiles in figure n.

b) Write an explicit formula for the number of tiles in figure n. This will require some creative thinking. Drawing pictures should help as well. Don't be afraid of trial and error and check your conjectures. There is a pictorial hint at the end of this chapter (that shows one way to think about it) but try to get it on your own before looking. Also note that there is more than one way to determine this formula.

2.2 Extra Practice

1)

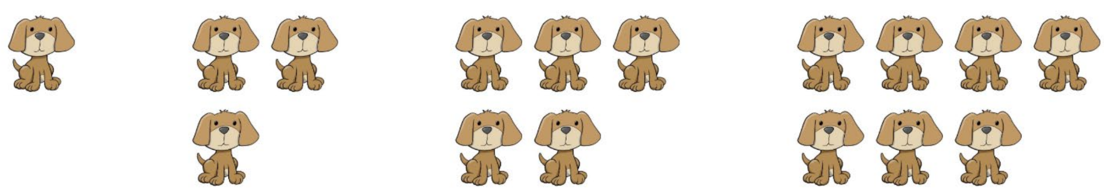


a. Describe the patterns in words. Be as specific as you can

b. Determine a recursive formula for the number of objects in Figure n

c. Determine an explicit formula for the number of objects in Figure n

2) Figure 1 Figure 2 Figure 3 Figure 4

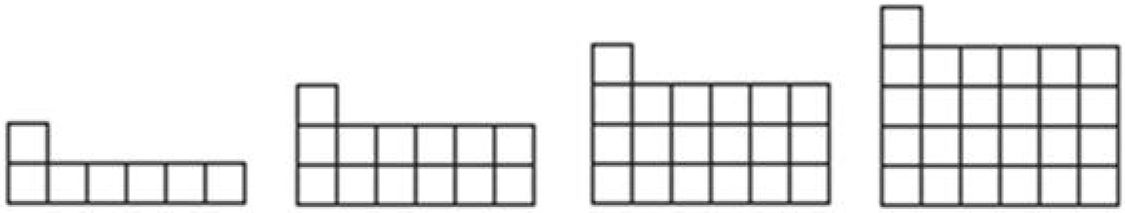


a. Describe the patterns in words. Be as specific as you can

b. Determine a recursive formula for the number of objects in Figure n

c. Determine an explicit formula for the number of objects in Figure n

3) Figure 1 Figure 2 Figure 3 Figure 4

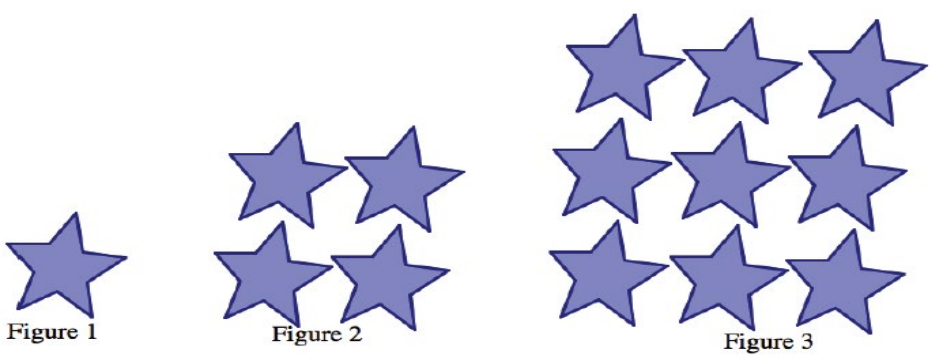


a. Describe the patterns in words. Be as specific as you can

b. Determine a recursive formula for the number of objects in Figure n

c. Determine an explicit formula for the number of objects in Figure n

4)



a. Describe the patterns in words. Be as specific as you can

b. Determine a recursive formula for the number of objects in Figure n

c. Determine an explicit formula for the number of objects in Figure n

2.2 Exercises

1) Answer each question below for the following pattern. Show and/or explain how you got your answers. Assume each tile is 1cm by 1cm.

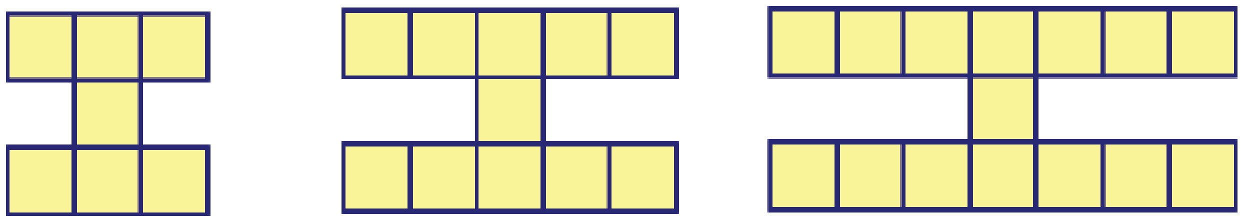


Figure 1 Figure 2 Figure 3

a) Write a recursive formula for the number of tiles in the nth figure.

b) Write an explicit formula for the number of tiles in the nth figure.

c) Write a recursive formula for the perimeter of the nth figure.

d) Write an explicit formula for the perimeter of the nth figure.

2) Answer each question below for the following pattern. Show and/or explain how you got your answers. Assume each tile is 1cm by 1cm.

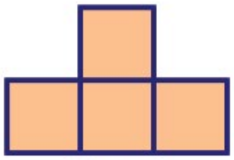


Figure 1

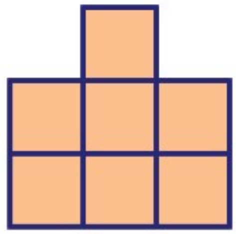


Figure 2

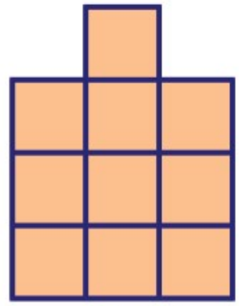


Figure 3

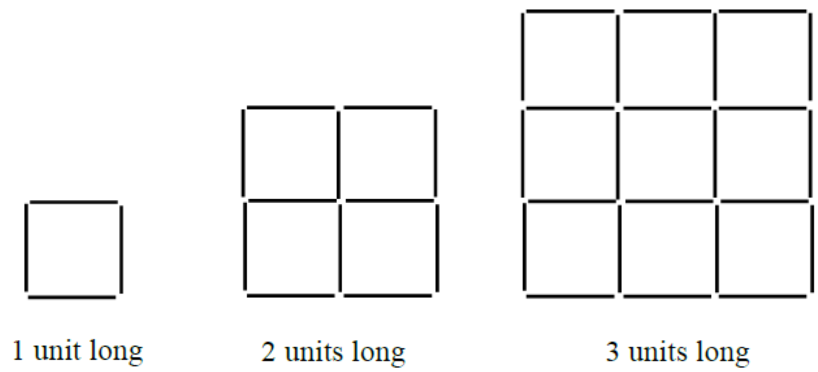
a) Write a recursive formula for the number of tiles in the nth figure.

b) Write an explicit formula for the number of tiles in the nth figure.

c) Write a recursive formula for the perimeter of the nth figure.

d) Write an explicit formula for the perimeter of the nth figure.

3) Consider the pattern shown below. Observe that the square grid that is 3 units long is made from 24 unit-long segments.



a) How many segments are required to make a square grid that is 10 units long?

b) Write a recursive formula for the number of segments required to make a square grid that is n units long. Show and/or explain how you got your answer.

c) Write an explicit formula for the number of segments required to make a square grid that is n units long. Show and/or explain how you got your answer.

For more practice with growing patterns go to https://www.visualpatterns.org/ and try to find recursive and explicit formulas for some of the patterns.

2.3 Sequences

In this section we look at number patterns which are often called ***sequences*** in mathematics. We call each number in the sequence a **"term"** (e.g. the first number in the sequence is called the "first term" and the second number in the sequence is called the "second term" and so on). Just as with the growing patterns, the goal here will be to describe the pattern in such a way that will enable us to find subsequent terms. We also take this opportunity to include terms with decimals and fractions which is always good practice!

Pre-Class Work

Watch the two videos below about sequences and respond to the prompts that follow.

1) Arithmetic and Geometric Sequences

2) The following video focuses on recursive patterns and includes arithmetic, geometric and other types of sequences: https://www.youtube.com/watch?v=4LfJ8rNZucA

Prompts to respond to for pre-class work:

1) Describe in your own words how you can tell what an arithmetic sequence is as if you were explaining it to an elementary school student. In doing so, create your own arithmetic sequence (not one from the videos) to illustrate your explanation.

2) Describe in your own words how you can tell what a geometric sequence is as if you were explaining it to an elementary school student. In doing so, create your own geometric sequence (not one from the videos) to illustrate your explanation.

Example 2.3.1

Consider the following sequences and describe the pattern of what is happening from one term to the next. Then determine the next three terms of each sequence.

a) 14, 16.5, 19, . . .

b) 80, 160, 320, . . .

c) 8, 5, 2, . . .

d) $\frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3} \ldots$ (hint: first rewrite to make all denominators 12)

e) $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$

f) –972, 2916, -8748, . . .

In the previous example we addressed two main types of sequences, one where we add or subtract to get the next term(s) and one where we multiply or divide to get the next term(s). These are common types of sequences that we define below.

Arithmetic Sequence: A sequence in which the same number is added to each term to get the next term. The number that is being added is called the ***common difference***, denoted ***d***.

Note the common difference may be negative.

Which sequences from Example 2.3.1 are arithmetic? What are the common differences in each of these sequences? **Record your answers below.**

Observe that we can find the common difference by taking any term and subtracting the previous term. Hence the name "common difference".

The growing patterns from the previous section corresponded to arithmetic sequences (discuss)

Geometric Sequence: A sequence in which the same number is multiplied by each term to get the next term. The number that we are multiplying by is called the ***common ratio***, denoted ***r***.

Note that if we are dividing by the same number to get to the next term in a sequence, this would also be a geometric sequence since dividing by a number is the same as multiplying by its reciprocal.

Which sequences Example 2.3.1 are geometric? What are the common ratios in each of these sequences? **Record your answers below.**

Observe that we can find the common ratio by taking any term and dividing by the previous term. Hence the name "common ratio".

Subscript Notation

There is a formal notation that is used when referring to the terms of a sequence. It is called *subscript notation* and we introduce it in an example below.

We denote the first term of a sequence as a_1 which is read "a sub one", short for "*a* with subscript one". Similarly, the second term of a sequence is denoted a_2 which is read "a sub 2" .

Consider the following sequence: 4, 6.5, 9, 11.5, …

$a_1 = 4$ (read "a sub 1 equals 4" meaning "the first term is 4")

$a_2 = 6.5$ (read "a sub 2 equals 6.5" meaning "the second term is 6.5")

$a_3 = 9$ (read "a sub 3 equals 9" meaning "the third term is 9")

and so on…

So the subscript tells us what term we are on. **In general a_n represents the *n*th term of a sequence. Observe that the term before a_n is represented by a_{n-1}**. We will use this when expressing rules using subscript notation.

Below we adapt our definition of recursive and explicit formulas/rules from the previous section to sequences and focus on expressing them using subscript notation.

A **recursive rule** defines a sequence by…

1) Identifying the first term
2) Giving a rule that describes each subsequent term using previous terms in the sequence (**nth term = something involving the (n-1)th term**)

Illustration

Once again consider the sequence 4, 6.5, 9, 11.5, …

Recursive rule in words	Recursive rule using subscript notation
The first term = 4 The nth term = The (n-1)th term + 2.5 For n = 2,3,4,...	$\boldsymbol{a_1 = 4}$ $\boldsymbol{a_n = a_{n-1} + 2.5}$ for *n = 2,3,4,...*

Below we check this recursive rule $\boldsymbol{a_n = a_{n-1} + 2.5}$ *for n = 2,3,4* to see how it works to generate the desired sequence: 4, 6.5, 9, 11.5, …

n = 1: $\boldsymbol{a_1 = 4}$

n = 2: $\boldsymbol{a_2 = a_{2-1} + 2.5 = a_1 + 2.5 = 4 + 2.5 = 6.5}$

n = 3: $\boldsymbol{a_3 = a_{3-1} + 2.5 = a_2 + 2.5 = 6.5 + 2.5 = 9}$

n = 4: $\boldsymbol{a_4 = a_{4-1} + 2.5 = a_3 + 2.5 = 9 + 2.5 = 11.5}$

It works!

Now let us recall how to form an explicit formula/rule…

An **explicit formula/rule** defines a sequence by giving a formula for the nth term without using previous terms (**nth term = something involving n only**).

Recall what we learned about the relationship between explicit and recursive rules in the growing patterns section (discuss). We use this to determine an explicit rule for 4, 6.5, 9, 11.5, … (using subscript notation) as follows:

Explicit formula/rule for 4, 6.5, 9, 11.5, **…** is $\boldsymbol{a_n = 4 + 2.5(n-1) = 2.5n + 1.5}$

Below we test this formula $\boldsymbol{a_n = 2.5n + 1.5}$ for *n = 2,3 and 4*

to see how it works to generate the desired sequence: 4, 6.5, 9, 11.5, …

n = 1: $\boldsymbol{a_1 = 2.5(1) + 1.5 = 4}$

n = 2: $\boldsymbol{a_2 = 2.5(2) + 1.5 = 5 + 1.5 = 6.5}$

n = 3: $\boldsymbol{a_3 = 2.5(3) + 1.5 = 7.5 + 1.5 = 9}$

n = 4: $\boldsymbol{a_4 = 2.5(4) + 1.5 = 10 + 1.5 = 11.5}$

It works!

Example 2.3.2

Consider the following geometric sequence from Example 2.3.1:

80, 160, 320, . . .

a) Write a recursive rule using subscript notation and check your answer by testing your rule for several values of n as we did in the previous illustration.

b) Write an explicit formula/rule for this sequence and check your answer by testing your rule for several values of n as we did in the previous illustration.

c) Describe the relationship between the recursive rule and the explicit formula/rule for this sequence.

Example 2.3.3

Recall the following sequences from Example 2.3.1. Write recursive and explicit formulas/rules for each using subscript notation. Use your explicit rule to find the 20th term of the sequence.

a) 8, 5, 2, . . .

Recursive rule:

Explicit formula/rule:

20th term:

b) $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$

Recursive rule:

Explicit formula/rule:

20th term:

c) –972, 2916, -8748, . . .

Recursive rule:

Explicit formula/rule:

20th term:

Example 2.3.4

Consider an arithmetic sequence with first term 8 and fifth term 20. Find the 100th term.

Applications of Arithmetic and Geometric Sequences

Example 2.3.5

The population of Utopia is predicted to increase by 1200 each year for the next 20 years. If the population is 44,000 now, how much will it be in 15 years?

Example 2.3.6

Three bacteria were placed in a petri dish. The number of bacteria quadruples every hour. There are now 196,608 bacteria in the dish. How many hours have passed since the original bacteria were placed in the dish? There are a few different ways to approach this problem. Work with a partner.

2.3 Exercises

For # 1 - 8, determine if each of the following sequences is arithmetic or geometric. Then write recursive and explicit formulas/rules for each using subscript notation.

1) –10, –20, –30, –40, …

2) 5, –15, 45, –135, …

3) 0.2, 3.0, 5.8, 8.6, …

4) $\frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{2}{3} \ldots$ (hint: find a common denominator first)

5) $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, \ldots$

6) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$

7) $3\frac{11}{12}, 3\frac{7}{12}, 3\frac{1}{4}, 2\frac{11}{12}, 2\frac{7}{12} \ldots$ (Observe $3\frac{1}{4} = 3\frac{4}{12}$)

8) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27} \ldots$

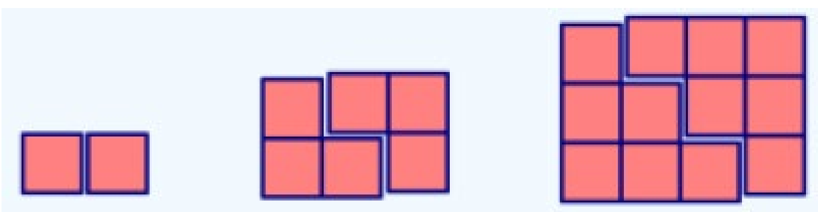
9) Miguel’s annual income has been increasing by the same amount every year. In the first year his income was $50,000. In the 6th year it will be $58,000. In what year will his income be $66,000.

10) One hundred kilograms of a toxic chemical was dumped illegally into a clean reservoir. A filter can remove 20% of the chemical still present each week (so that 80% of the previous amount remains). How much of the chemical will remain in the water after 20 weeks?

Chapter 2 Wrap Up

Problems based on patterns provide excellent experiences for students of all ages to describe patterns, make generalizations, and then make mathematical arguments. Problems like these can stem from a hands-on activity or process that the students can see and lead to an abstract generalization. The repeating and growing patterns in this chapter provided a foundation for the more abstract patterns we see in sequences. The use of the variable n highlights the importance of algebra in clearly and concisely describing patterns. The next chapter combines algebra and patterns into the concept of a *function* which relates two quantities. Explicit formulas for sequences are a specific type of function because they relate the position (n) of each term to the term itself. More details to come in chapter 3!

Hint for Example 2.2.6 (b)



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