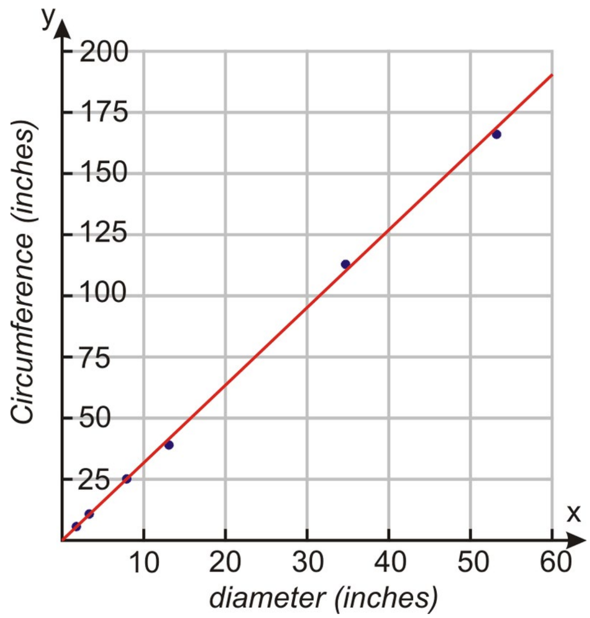
Chapter 4: LINEAR FUNCTIONS



Circumference as a Linear Function of Diameter by Andrew Gloag, Melissa Kramer and Anne Gloag, licensed by CK-12

In this chapter…

4.1 Introduction to Linear Functions
4.2 Linear Models
4.3 Linear Systems

Elementary school activities may provide opportunities for children to observe linear functions early on. For example, students could plant seeds and record daily heights in a table. If the plant grows the same amount each day, this is linear and if it grows by different amounts each day, then it is non-linear. They are subsequently introduced to input/output tables with linear relationships.

Later in elementary school students explore the properties of such relationships, which we will study in this section. Learning about rate of change prepares them for the concept of slope, which is essential to identifying linear functions.

According to the **National Council of Teachers of Mathematics (NCTM) Principles and Standards** for School Mathematics…

In prekindergarten through grade 2 all students should…

- describe quantitative change, such as a student's growing two inches in one year.

In grades 3–5 all students should…

- investigation how a change in one variable relates to a change in a second variable
- identify and describe situations with constant or varying rates of change and compare

In grades 6–8 all students should…

- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations
- use graphs to analyze the nature of changes in quantities in linear relationships.

In the **Massachusetts Mathematics Curriculum Framework**…

Grade 8

8.F.A. Define, evaluate, and compare functions.

3. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

8.F.B. Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities…… Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

4.1 Introduction to Linear Functions

Pre-class work

Go to Linear equations and graphs: FAQ in Khan Academy and complete the exercises corresponding to the questions below. Make note of questions you have to ask about in class.

What is slope?

What are horizontal and vertical lines?

What are x and y intercepts?

If you need a review to successfully complete these exercises, watch one or more of the videos below and/or read the following review.

Shorter one: https://www.youtube.com/watch?v=RqHMpZzG-FM (~7 min.)

Longer one: https://www.youtube.com/watch?v=Ft2_QtXAnh8 (~30 min.)

Video on x and y intercepts

Review of Linear Equations

Recall the slope of the line through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example: Find the equation of the line through the points (7, 5) and (-7, -1) and then graph it.

Solution: The slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{-7-7} = \frac{-6}{-14} = \frac{3}{7}$. So we know that our equation looks like $y = \frac{3}{7}x + b$. Now we just need to find b which we can do by plugging in either of the given points on the line. Let's use (7, 5). So we plug in x = 7 and y = 5 and solve for b as follows:

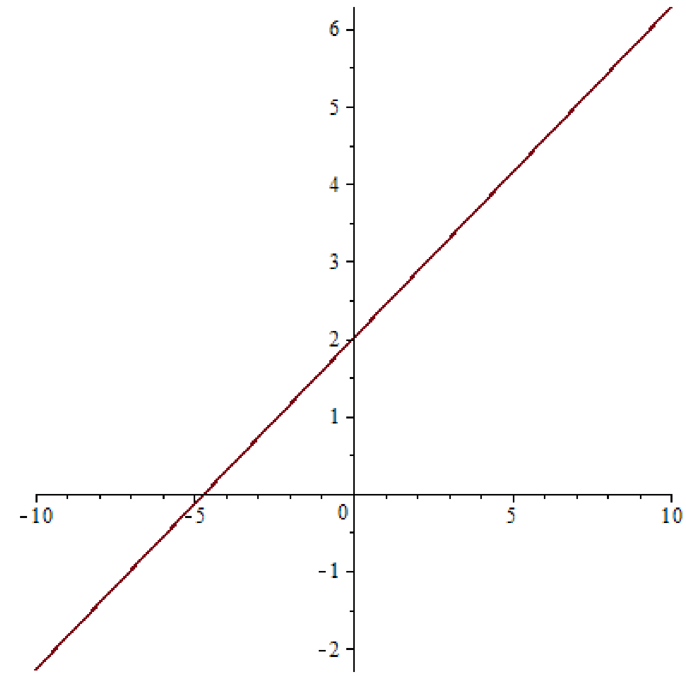
$5 = \frac{3}{7}(7) + b$ so $5 = 3 + b$ and b = 2. So our final equation is $y = \frac{3}{7}x + 2$.

Now let's graph it!

There are many ways in which we can do this. One way is to simply plot the two points given and draw a straight line through them. Another way is to first plot the y-intercept (0,2) and then use the slope to get another point as follows:

$\frac{change\_in\_y}{change\_in\_x} = \frac{rise}{run} = \frac{3}{7}$ so, starting at (0,2) we go up 3 then to the right 7 and plot another point on the line.

Observe that $\frac{3}{7} = \frac{-3}{-7}$ so we can also go down 3 and to the left 7 to plot another point on the line. Either way, once we have two points, we can simply connect them with a straight line to get the graph as shown.



Intercepts are important points on a graph. The y-intercept is where the graph intersects the y-axis and the x-intercept is where the graph intersects the x-axis.

We can see the y-intercept of the graph above is (0,2) but what is the x-intercept? Looking at the graph we see that it is close to (-5, 0) but what is it exactly?

Observe y = 0 for any x-intercept so we can find the x-intercept by setting y = 0 in the equation we found, and solving for x as follows:

$$0 = \frac{3}{7}x + 2$$

$$-2 = \frac{3}{7}x$$

$$x = (-2)\frac{7}{3} = \frac{-14}{3} \approx -4.7$$

So the x-intercept of the line above is $\left(\frac{-14}{3}, 0\right) \approx (-4.7, 0)$.

Linear Functions and Growing Patterns

Recall the growing pattern below.

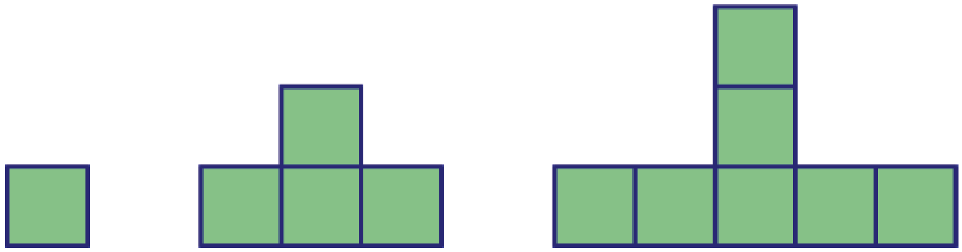


Figure 1　　Figure 2　　Figure 3

Figure number (n) (input)	1	2	3	4	5	…	n
number of tiles (T) (output)	1	4	7	10	13	…	3n - 2

The number of tiles increases by the same amount, 3, from one figure to the next. In particular, as the figure number increases by 1, the number of tiles increases by 3. So the rate of change of the total number of tiles is 3 (for each unit increase in the figure number). This rate of change/slope is constant, which is what makes this relationship (between the figure number and the number of tiles in the figure) a linear function (the graph forms a line).

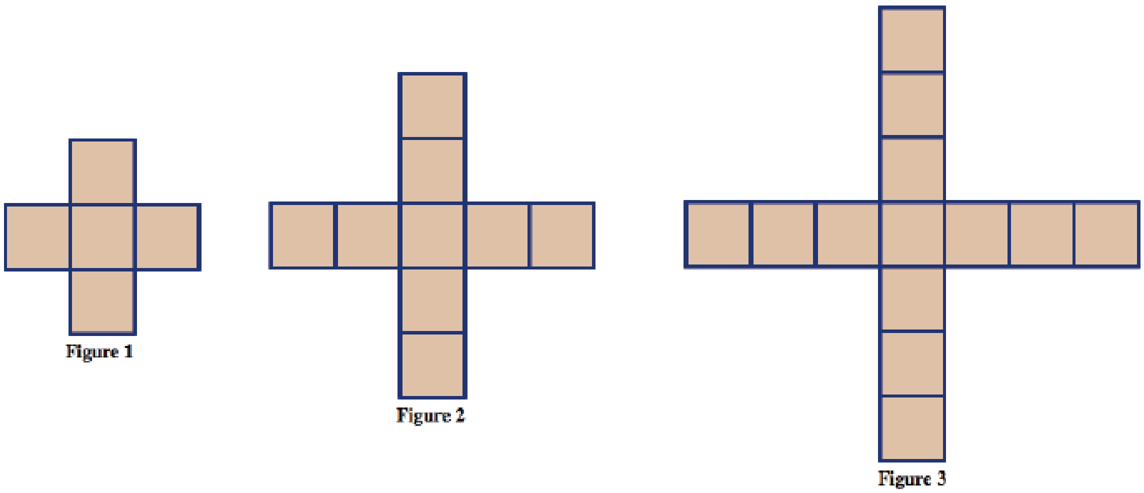
We may write this relationship as $y = 3x - 2$, where x represents the figure number and y represents the number of tiles in figure x. Now this should look more familiar to you as a linear function from your past knowledge.

In general, when we establish a relationship between two quantities in which the rate of change is constant, we categorize that relationship as linear. Let us review the formal definition of a linear function.

A **linear function** is one that can be written in the form $y = mx + b$ where m and b are constant. m is the **slope** and (0,b) is the **y-intercept** of the line (that is, where the graph intersects the y-axis).

Example 4.1.1

Consider the growing pattern below.



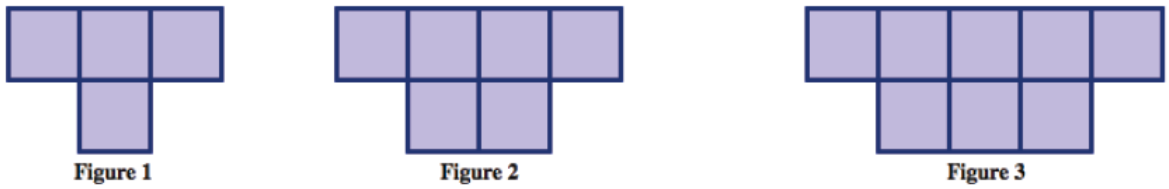
a) Determine a linear equation in the form y = mx + b, where x represents the figure number and y represents the number of tiles in figure x.

b) State the rate of change/slope and what it means in terms of the pattern.

c) State the y-intercept and what it means in terms of the pattern.

Example 4.1.2

Consider the growing pattern below.



d) Determine a linear equation in the form y = mx + b, where x represents the figure number and y represents the number of tiles in figure x.

e) State the rate of change/slope and what it means in terms of the pattern.

f) State the y-intercept and what it means in terms of the pattern.

Example 4.1.3

Connection with Arithmetic Sequences: [Subsection: Arithmetic Sequences](). Read and work through the corresponding "checkpoint" exercises.

4.1 Exercises

From CourseHero https://www.coursehero.com/file/85655103/Linear-Functions-HWdoc/

Slope-Intercept Form $y = mx + b$

Determine the slope and *y*-intercept for each graph. Write the equation for the graph in slope-intercept form

1.

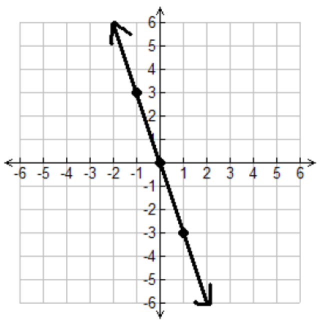
slope	
y-intercept	
equation	

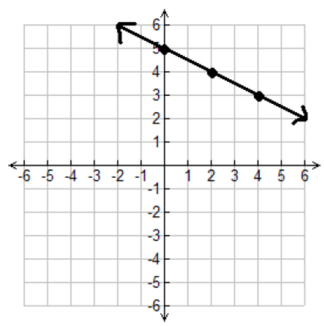
2.

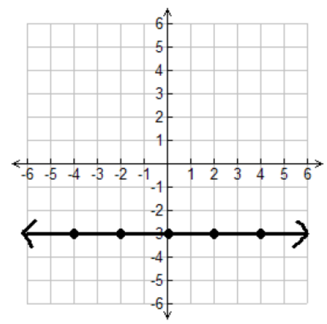
slope	
y-intercept	
equation	

3.

slope	
y-intercept	
equation	







Write each equation in slope – intercept form, identify the slope and y-intercept, then graph each equation. Clearly mark at least three points on each line. Show your work. Label the x and y-axis.

4. $y = 6$

____

slope __________

y-intercept (,)

5. $x - y = 4$

____

slope ___________

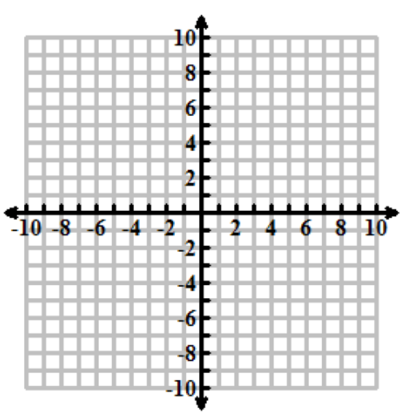
y-intercept (,)

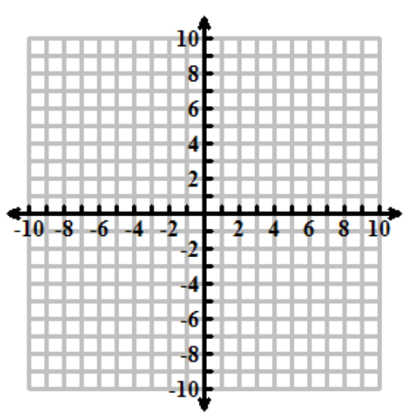
6. $5x - 3y = 24$

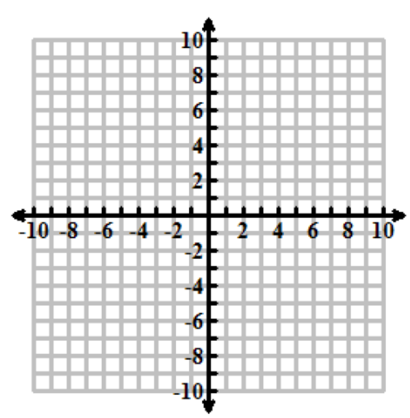
____

slope __________

y-intercept (,)







7. $2x + y = 0$

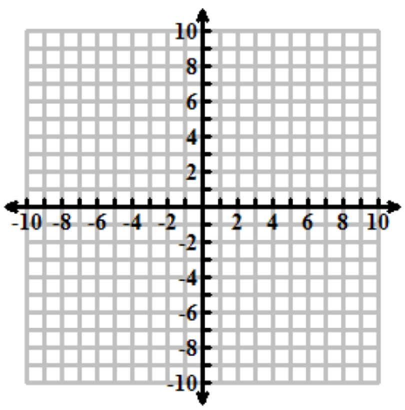
8. $-x + 4y = 4$

9. $x = -4$

____

slope _________

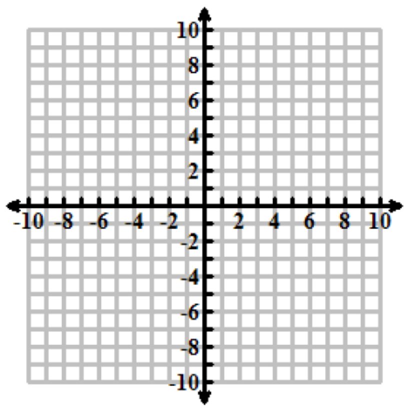
y-intercept (,   )



____

slope __________

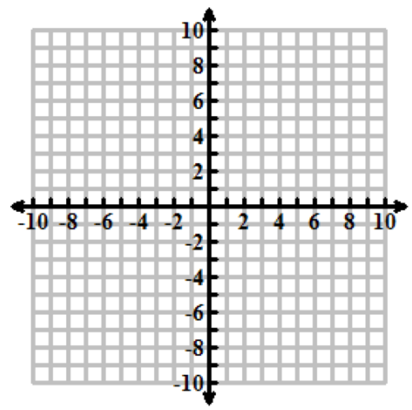
y-intercept (,   )



____

slope _________

y-intercept (,   )



Determine the slope (rate of change) and the y-intercept (start value) for each table. Write an equation in slope-intercept form.

10.

x	*y*
−2	0
0	10
2	20
4	30
6	40

Slope: ______
y-intercept: (,   )
Equation: ___________

11.

x	*y*
−2	6
1	3
3	1
4	0
10	−6

Slope: ______
y-intercept: (,   )
Equation: ___________

12.

x	*y*
5	6.8
6	7
7	7.2
8	7.4
9	7.6

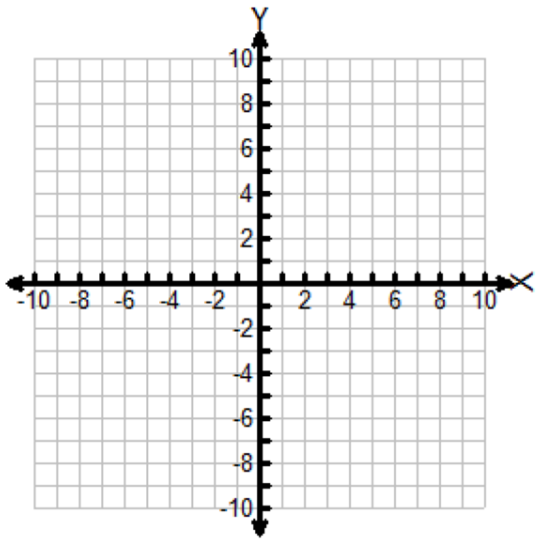
Slope: ______
y-intercept: (,   )
Equation: ___________

Point-Slope Form $\boldsymbol{y - y_1 = m(x - x_1)}$

Write an equation of the line in point-slope form through the given point and with the given slope *m*. Then graph the line on the coordinate plane. Clearly show 3 points on the line.

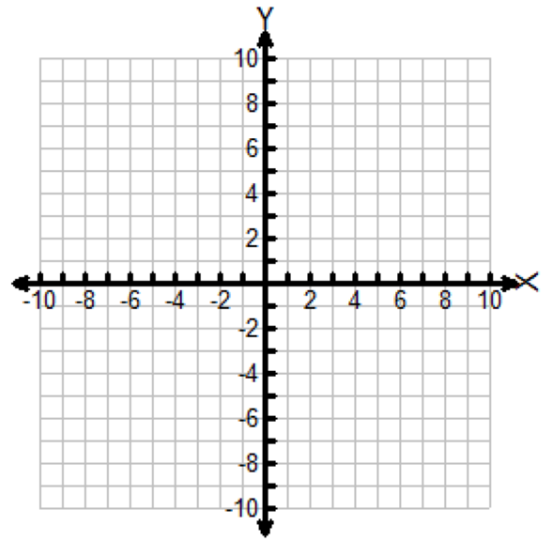
13. (-3, -5); *m* = -2

Equation: ___________________________



14. (0, -3); *m* = $-\frac{1}{2}$

Equation: ___________________________

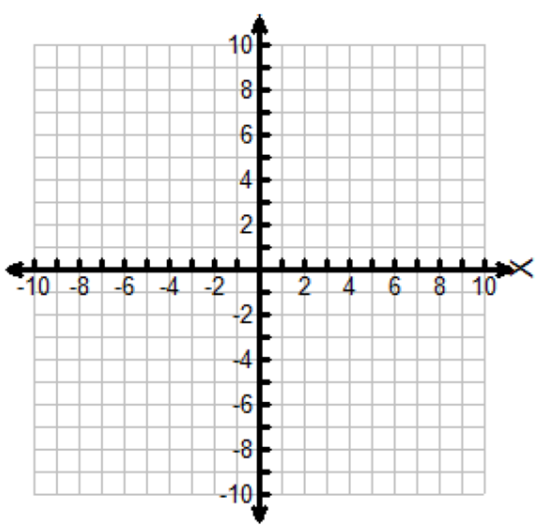


Identify the slope and the point on the line then graph each equation. Clearly show 3 points on the line.

15. $y - 2 = 2(x + 3)$

Slope: _____________________

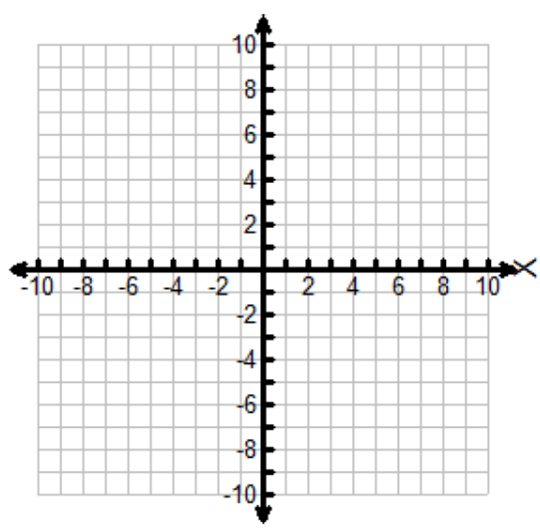
Point: _____________________



16. $y + 5 = -\frac{3}{4}(x - 4)$

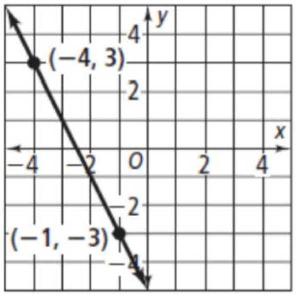
Slope: _____________________

Point: _____________________



Write an equation in point-slope form for the line.

17.



Write an equation in point-slope form of the line through the given points. Then write the equation in slope-intercept form. (slope-intercept y = mx + b)

18. (0, 4), (1, 7)

Slope =

Point – slope:

____

Slope-intercept:

____

19. (−2, −9), (3, 11)

Slope =

Point – slope:

____

Slope-intercept:

____

20. (2, 2), (5, -4)

Slope =

Point – slope:

____

Slope-intercept:

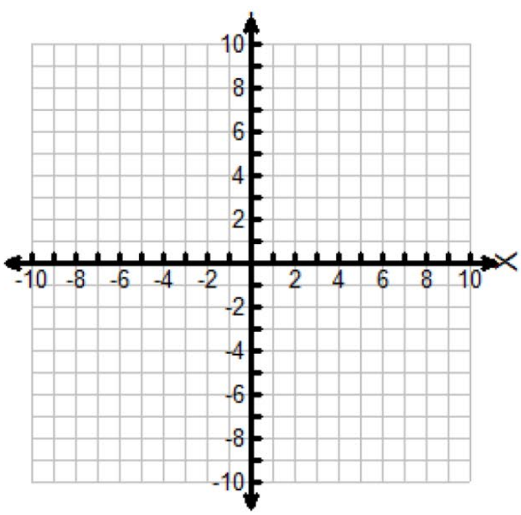
____

Standard Form $Ax + By = C$

Graph each equation using the x- and y-intercepts. Show work for finding the intercepts.

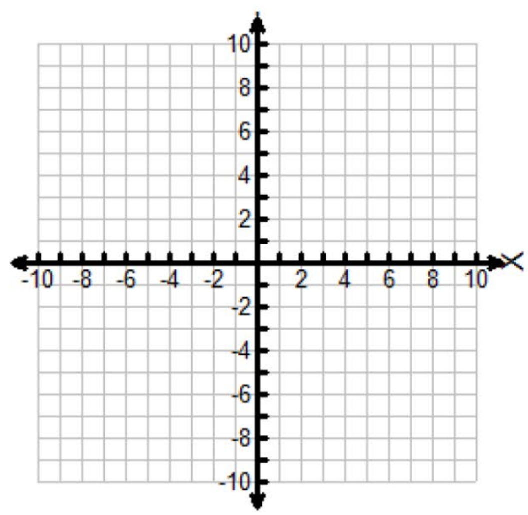
21. $-5x + y = -10$

x-int. (____, ____) y-int. (____, ____)



22. $-3x - 6y = 12$

x-int. (____, ____) y-int. (____, ____)



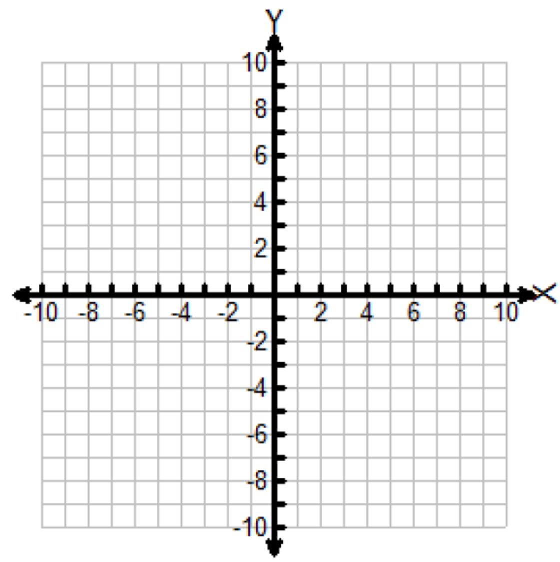
For each equation, identify whether its graph is a ***horizontal*** or a ***vertical*** line, state the slope, then draw graph. Graph all four lines on the same coordinate plane. Label each line with its equation.

23. y = -5 horizontal/vertical slope: _________

24. x = -4 horizontal/vertical slope: _________

25. x = 7 horizontal/vertical slope: _________

26. y = 8 horizontal/vertical slope: _________



Write each equation in standard form using integers.

27. $y - 4 = 5(x - 8)$

28. $y = x - 4$

29. $y = \frac{-3}{5}x + 2$

Write an equation that passes through the pair of points in point-slope form, slope-intercept form and standard form using integers.

30. (4, -2), (5, -4)

Slope=

Point-slope: ____________________

Slope-intercept: ___________________

Standard: _________________________

31. (-5, -5), (10, 4)

Slope=

Point-slope: ______________________

Slope-intercept: ____________________

Standard: ___________________________

4.2 Linear Models

Pre-Class Work

Work through the problems at the following links in Kahn Academy, record your work and bring it to the next class for discussion.

Relating linear contexts to graph features

Graphing linear relationships-word problems

Comparing linear rates-word problems

Many real world scenarios involve quantities that increase or decrease at a constant rate and hence may be modeled by linear functions. In this section we focus on applications of linear functions and work through several modeling problems.

Example 4.2.1

Franco plants a dozen corn seedlings, each 6 inches tall. With plenty of water and sunlight, they will grow approximately 2 inches per day.

a) Complete the table of values for the height, *h*, of the seedlings after *t* days.

t	0	5	10	15	20
h					

b) What is the (average) rate of change of the height of the plants from 0 to 5 days? What about from 5 - 10 days? What about 10 - 20 days?

c) Write an equation for the height *h* of the seedlings in terms of the number *t* of days since they were planted.

d) Graph the equation with t on the horizontal axis and h on the vertical axis.

Example 4.2.1 illustrates a linear function. Recall…

A **linear function** is a function in which the rate of change/slope is constant.

The **rate of change/slope** is the amount that the output variable changes per unit change in the input variable.

In example 4.2.1 we view the input as time t in days and the output as height h in inches. Here, the slope is 2 because the height changes by 2 inches per day.

Recall the different forms of linear functions (with input variable x and output variable y)

Slope-intercept form: $y = mx + b$ where m is the slope and (0,b) is the y-intercept.

Standard or general form: $ax + by = c$ where a, b and c are constants

Point-Slope form: $y - k = m(x - h)$ where m is the slope and (h, k) is a point on the line

In general, for any function, the **y-intercept** is the point on the graph where x = 0, that is, where the graph intersects the y-axis and the **x-intercept** is the point on the graph where y = 0, that is, where the graph crosses the x-axis.

Example 4.2.2

Consider a line that goes through the points (7, 5) and (-7, -1)

a) Find an equation for this line in each of the three forms given above.

b) Find the x and y intercepts of the line and graph it below, showing the intercepts.

Example 4.2.3

Suppose Keng has a job raking leaves where he gets paid $5.00 an hour for raking and an additional $3.00 for hauling the leaves to the curb for pick up.

a) How much will he get paid if he rakes for 2 hours and hauls the leaves? Explain your answer.

b) How long must he rake to earn $19.00 including the hauling fee? Explain your answer.

c) Represent the relationship between hours worked (x) and money earned (y), including hauling charge as an equation, a table and as a graph.

d) What is the slope and what does it tell us about Keng's earnings? Be specific and include units in your explanation.

e) What is the y-intercept and what does it tell us about Keng's earnings? Be specific and include units in your explanation.

Example 4.2.4

Delbert must increase his daily potassium intake by 1800 mg. He decides to eat a combination of figs and bananas. There are 9 mg of potassium per gram of fig, and 4 mg of potassium per gram of banana.

a) Write an expression that represents the amount of potassium in *x* grams of fig.

b) Write an expression that represents the amount of potassium in *y* grams of banana.

c) Write an equation, in standard form, that relates the number of grams of fig (x) and the number of grams of banana (y) Delbert needs to consume in order to get 1800 mg of potassium.

d) Find the x and y intercepts of this equation and sketch the graph.

e) What do each of the intercepts tell us about Delbert's diet?

x-intercept:

y-intercept:

Example 4.2.5

According to one study, 27.5% of high school seniors reported vaping nicotine in 2018 and approximately 39.5% of high school seniors reported vaping nicotine in 2021.

a) Use this data to write a linear function with y as the percent of teens who vaped nicotine x years after 2010.

b) Use your function to predict the percent of high school seniors who vaped nicotine in 2022.

c) Find the x-intercept of the graph of your equation (you may approximate to the nearest tenth). Explain what this point tells us about the percent of high school seniors vaping.

d) What is the slope of your function? Explain what this number tells us about the percent of high school seniors vaping.

e) Sketch the function. Be sure to label the axes with the corresponding values they represent in terms of the application.

4.2 Exercises

1) Postage for a package is \$3 for the first pound and \$2 for each additional pound.

   a) Model the postage price, y , as a function of weight, x, in pounds using a linear equation.

   b) What is the slope and what does it tell us about postage price? Be specific and include units in your explanation.

   c) What is the y-intercept and what does it tell us about postage price? Be specific and include units in your explanation.

2) The relationship between degrees Fahrenheit (F) and degrees Celsius (C) is linear. Water freezes at 0°C and 32°F. Water boils at 100°C and 212°F.

   a) Write an equation that expresses Fahrenheit temperature F as a linear function of Celsius temperature C.

   b) Determine the slope and explain what it tell us about degrees.

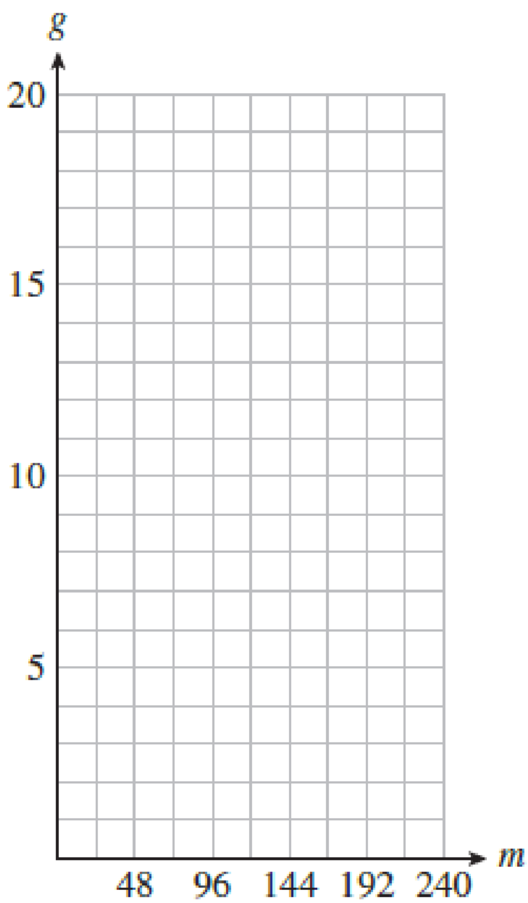
3) Kisha's camper has a 20 gallon gas tank and she gets 12 miles to the gallon (that is, she uses 1/12 gallon per mile).

a) Complete the table of values for the amount of gas, g, left in Kisha's tank after driving m miles, assuming she starts with a full tank.

m	0	48	96	144	192
g					

b) Write an equation that expresses the amount of gas, g, left in Kisha's tank in terms of the number of miles, m, she has driven.

c) Graph the equation below



d) How much gas will Kisha use between when her odometer reads 96 miles and when the odometer reads 144 miles? Illustrate this on the graph.

e) If Kisha has less than 5 gallons of gas left, how many miles has she driven? Illustrate this on the graph.

4) Mayuri works as a waiter in a restaurant. He earns $1,400 per month as a base salary, plus tips averaging 15% of the total cost of the meals he served that month.

   a) Let S represent how much Mayuri earns in one month and C represent the total monthly meal cost. Write a linear equation expressing S in terms of C.

   b) Determine the slope of the function and explain what this number tells us about Mayuri's monthly earnings.

   c) Determine the y-intercept of the equation and explain what this point tells us about Mayuri's monthly earnings.

5) Consider the linear equation $2x + 5y = 30$. Write a story problem that would result in this equation (something that gives meaning to the numbers 2, 5 and 30). Then, state what the variables x and y represent in terms of your story problem scenario.

6) There is a population of 200 tigers in a national park. Unfortunately, they are being illegally poached at the rate of 8 tigers per year. Assume the population is otherwise unchanging:

   a) Write a linear model for the tiger population, P, in t years.

   b) Graph your linear model from part (a).

   c) Determine the t-intercept and explain what it tells us about the tiger population.

   d) Determine the P-intercept and explain what it tells us about the tiger population.

   e) Find how many years would it take for these tigers to become extinct in this park?

   f) How many years would it take to reduce the number of tigers in this park to half of the current population?

4.2 Extra Practice

For numbers 1 - 6 write an equation, in slope-intercept form, to model each situation. Clearly describe what each of your variables represents.

1) You rent a bicycle for $20 plus $2 per hour.

2) An auto repair shop charges $50 plus $25 per hour.

3) A candle is 6 inches tall and burns at a rate of ½ inch per hour.

4) The temperature is 15º and is expected to fall 2º each hour during the night.

5) A computer technician charges $75 for a consultation plus $35 per hour.

6) The population of Pine Bluff is 6791 and is decreasing at the rate of 7 per year.

7) In 1995, Orlando, Florida population was about 175,000. At that time, the population was growing at a rate of about 2000 per year.

a) Write an equation, in slope-intercept form to find Orlando's population for any year.

b) Use your model from part (a) to predict what Orlando's population will be in 2010.

8) Couples are marrying later in life. The median age of men who married for the first time in 1970 was 23.2. In 1998, the median age of men marrying for the first time was 26.7.

a) Write an equation, in slope-intercept form to predict the median age (M) of men marrying for the first time, for any year t.

b) Use the equation from part (a) to predict the median age of men who marry for the first time in 2005.

9) The cost for 7 dance lessons is \$82. The cost for 11 lessons is \$122.

a) Write a linear equation in slope-intercept form, to find the total cost C for L lessons.

b) Use the equation from part (a) to find the cost of 4 lessons.

10) It is 76º F at the 6000-foot level of a mountain, and 49º F at the 12,000-foot level of the mountain.

a) Write a linear equation, in slope-intercept form, to find the temperature T at an elevation E on the mountain, where E is in **thousands of feet**.

b) Use the equation from part (a) to predict the temperature at an elevation of 20,000 feet.

11) Between 1990 and 1999, the number of movie screens in the United States increased by about 1500 each year. In 1996, there were 29,690 movie screens.

a) Write an equation of a line, in slope-intercept form, to find y, the total number of screens, x years after 1990.

b) Use the equation from part (a) to predict the number of movie screens in the United States in 2005.

12) A construction company charges $15 per hour for debris removal, plus a one-time fee for the use of a trash dumpster. The total fee for 9 hours of service is $195.

a) Write an equation of a line, in slope-intercept form, to find the total fee y for any number of hours x.

b) What is the fee for the use of a trash dumpster for 5 hours?

13) The population of Jose's town in the year 1995 was 2400 people and the population in the year 2000 was 4000 people. Let x represent the number of years since 1995.

a) Write a linear equation, in slope-intercept form, that represents this data.

b) Use the equation from part (a) to predict the population in Jose's town in 2010.

14) Randy owns a computer store. In 1990, he sold 150 monitors. In 2000, he sold 900 monitors. Let x represent the number of years since 1990.

a) Write a linear equation, in slope-intercept form, that represents this data.

b) Use the equation from part (a) to predict the number of monitors Randy will sell in 2007.

15) Romi opened a gift shop in 1995 and closed it in 2000. In 1995, her inventory of stuffed animals was 350. In 2000, her inventory of stuffed animals was 0. Let x represent the number of years since 1995. Write an equation, in slope-intercept form, that represents that data.

Supplemental Material on Linear Models Section 1.1: Linear Models

4.3 Linear Systems

Pre-Class Work

Read and work through checkpoint exercises at the following link

Section 8.1: Systems of Linear Equations in Two Variables

When it says to use your calculator to graph, you may use the graphing calculator at https://www.desmos.com/

Example 4.3.1

A farmer has pigs and chickens. If she counts 14 heads and 40 legs total, how many of each animal does she have? Solve this problem in two ways as noted below. Share your solutions in your group and discuss how they compare/contrast.

a) Solve without using any variables and in a way that an elementary school student could understand. Draw pictures and explain your calculations. Clearly document any trial and error (guessing and checking) involved.

b) Set up two equations, with two variables, to represent the given information. Clearly state exactly what each variable represents. Then solve for the variables using the substitution or elimination method. Show your work below. If you need a refresher on these methods, refer to the pre-class work or the review after Example 4.3.2.

c) Solve the problem by graphing the corresponding equations. Sketch your graph and justify your answer below.

Example 4.3.2

Sandrine “accidentally” broke her piggy bank to find a combined total of 42 dimes and quarters. If the coins totaled \$8.25, how many dimes and how many quarters did she have in her piggy bank?

a) Solve without using any variables and in a way that an elementary school student could understand. Draw pictures and explain your calculations. Clearly document any trial and error (guessing and checking) involved.

b) Set up two equations, with two variables, to represent the given information. Clearly state exactly what each variable represents. Then solve for the variables using the substitution or elimination method. Show your work below. If you need a refresher on these methods, refer to the pre-class work or the review after this example.

c) Solve the problem by graphing the corresponding equations. Sketch your graph and justify your answer below.

Review of Linear Systems (with two equations)

A **2 by 2 linear system** (of equations) is a set of 2 linear equations with the same 2 variables.

Illustration: $x + 2y = 12$
$2x - 4y = 3$

A **solution** of a 2 by 2 linear system is an ordered pair of values for the variables that makes each equation in the system true. As shown in the pre-class work, three ways to solve such systems are…

1) Using the substitution method
2) Using the elimination method
3) Graphing

Below we review these three methods

Substitution Method Review/Refresher

Khan Academy Video (~10 min): https://www.youtube.com/watch?v=V7H1oUHXPkg

Summary of steps:

1) Solve one of the equations for one of the variables in terms of the other.
2) Substitute the expression resulting from step (1) into the second equation; doing so yields an equation with one variable only.
3) Solve the new equation.
4) Plug your result from step (3) into the equation from step (1) to find the other variable.

Example 4.3.3

Use the substitution method to solve the following systems of equations.

a)	b)
$4x+3y=96$ $x+y=27$	$2x-y=2$ $4x-4y=-22$

The method of substitution is convenient if one of the variables in the system has a coefficient of 1 or -1, because it is easy to solve for that variable. If none of the coefficients is 1 or -1, then the elimination method is usually more efficient.

Elimination Method Review/Refresher

Khan Academy Video (~12 min): https://www.youtube.com/watch?v=vA-55wZtLeE

Summary of the Steps:

1) Choose one of the variables to 'eliminate'. Multiply one or both equations by a suitable factor so that the coefficients of that variable are opposites in the equations. The factor for each equation may be different.
2) Add the two new equations together which will eliminate the variable chosen in step 1.
3) Solve the resulting equation for the remaining variable.
4) Substitute the value found in step 3 into either of the original equations and solve for the other variable.

Example 4.3.4

Use the elimination method to solve the following systems of equations.

a)	b)
$5x - 2y = -4$ $-6x + 3y = 5$	$3x - 4y = -11$ $2x + 6y = -3$

Graphing Method Review/Refresher

Khan Academy Video (~12 min): https://youtu.be/5a6zpfl50go

Summary of the Steps:

1) Graph the lines given by each equation on the same set of axes.
2) Find the intersection point(s) of the two lines, if they exist. This gives you your solution(s)

Example 4.3.5

Check your answers from Example 4.3.3 and 4.3.4 by graphing in Desmos. Record any discrepancies below as points for discussion.

In the examples above, each system has exactly one solution. In the examples below, we explore other scenarios.

Example 4.3.6

Solve each system below using substitution or elimination and then by graphing. Explain what you see.

a) $x + y = 5$
   $y = 4x$

b) $2x + 4y = 0$
   $x + 2y = 0$

c) $x = y$
   $x - y = 6$

Example 4.3.7

Graphically explain (in general) how a 2 by 2 linear system could have each of the following scenarios and then discuss how you can tell which scenario you are in by looking at the equations.

Exactly one solution:

No solutions:

An infinite number of solutions:

One may also solve such problems using numerical methods, logic and possibly some trial and error (guess and check) as we did in Examples 4.3.1 and 4.3.2.

The ability to solve linear systems in this manner depends on the level of complexity. For example, if the solution involves decimals or fractions that are not whole numbers, then it may be difficult, if not impossible to solve the system by trial and error. However, if the answers are relatively small whole numbers, it is feasible to solve such problems without knowing anything about algebra or graphing. Hence, such problems may be introduced in the elementary school classroom.

Let us look at another application at this level.

Example 4.3.8

Pablo paid $33 for 3 chocolate bars and 9 cookies. Brenda paid $48 for 12 chocolate bars and 8 cookies. Find the cost of one chocolate bar and the cost of one cookie. Assume each costs a whole dollar amount (no cents). Solve this problem in two ways as noted below.

a) Solve without using any variables in a way that an elementary school student could understand. Draw pictures and explain your calculations. Clearly document any trial and error (guessing and checking) involved.

b) Solve using the substitution or elimination method. Clearly state exactly what each variable represents.

4.3 Exercises

1) Solve the following linear systems using substitution or elimination. Show all your work.

   a)
   $$7x - 4y = 1$$
   $$3x + y = 14$$

   b)
   $$2x - 3y = -4$$
   $$5x + 2y = 9$$

2) Kwazi had 13 coins, some nickels and some dimes. If the total value of the coins is 75 cents, how many nickels and how many dimes does Kwazi have? Solve this problem in two ways as follows:

   a) Without using any variables and in a way that an elementary school student could understand. Draw pictures and explain your calculations. Clearly document any trial and error (guessing and checking) involved.

   b) Using the substitution or elimination method. Clearly state exactly what each variable represents.

3) A bicycle store has many disassembled bicycles and tricycles. Altogether, there are 43 wheels and 17 frames. If all the bicycles and tricycles were assembled, how many bicycles and how many tricycles would there be? Solve this problem in two ways as follows:

   a) Without using any variables and in a way that an elementary school student could understand. Draw pictures and explain your calculations. Clearly document any trial and error (guessing and checking) involved.

   b) Using the substitution or elimination method. Clearly state exactly what each variable represents.

4) Create three different 2 by 2 linear systems for each of the following scenarios:

   a) Exactly one solution

   b) No solutions

   c) An infinite number of solutions

5) Heidi weighs 4 pounds more than Grace. Grace weighs twice as much as Jen. The sum of all their weights is 58 pounds. How much does each person weigh?

Let G represent Grace's weight in pounds, H represent Heidi's weight in pounds and J represents Jen's weight in pounds. Write three equations that would help you solve this problem. Then try to solve it using a series of substitutions. Don't be afraid of some trial and error here. Show your work below.

4.3 Extra Practice

Solve the following coin problems using any method. Show your work.

1) Sholay has 600 quarters and dimes in her piggy bank which totals $123.75. How many quarters does she have?

2) A coin bank contains $17 in pennies and nickels. If there are 1140 coins in the bank, how many of them are nickels?

3) A jar of coins contains six times as many quarters as dimes. If the total amount of money is $28.80, how many quarters and dimes are in the jar?

4) Liani cleaned the coin fountain at the mall and found 20 coins consisting of nickels and quarters. Her collection totaled to $2.60. How many quarters did she find?

5) Paco has twelve more pennies than he has nickels. All together he has $2.94. How many of each coin does he have?

6) Johan has 14 coins that have a total value of $2.30. The coins are nickels and quarters. How many of each coin does he have?

7) Laureen has $0.95 in dimes and nickels. She has a total of 11 coins. How many of each coin does she have?

8) Dana has 21 coins totaling $3.45. If he only has dimes and quarters, how many of each type of coin does he have?

9) A coin collector has 31 dimes and nickels with a total value of $2.40. How many of each coin do they have?

Chapter 4 Wrap Up

In this chapter we studied linear functions and systems and how they are modeled in real life scenarios. We saw how certain growing patterns, especially those taught in the elementary classroom, are closely linked to linear functions and how these ideas evolve into the ability to solve more complex word problems. Linear systems were also used to solve elementary problems as well as more complex problems requiring the use of algebra.

Hopefully at this point you realize the strong connections between the arithmetic in the elementary classroom and how these ideas grow up into the use of algebra. In the subsequent chapters we will study problems that involve nonlinear functions and applications thereof.